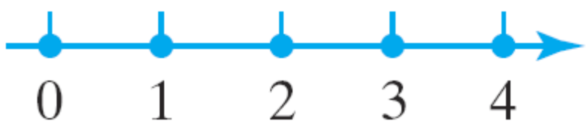
6.1: Discrete Random Variables

Discrete and Continuous Random Variables

A **random variable** is a numerical measure of the outcome from a probability experiment, so its value is determined by chance. Random variables are denoted using letters such as X.

A **discrete random variable** has either a finite or countable number of values. The values of a discrete random variable can be plotted on a number line with space between each point.



A **continuous random variable** has infinitely many values. The values of a continuous random variable can be plotted on a line in an uninterrupted fashion.

Example

Determine whether the following random variables are discrete or continuous. State possible values for the random variable.

(a) The number of light bulbs that burn out in a room of 10 light bulbs in the next year.

(b) The number of leaves on a randomly selected oak tree.

(c) The length of time between calls to 911.

Identify Discrete Probability Distributions

A **probability distribution** provides the possible values of the random variable X and their corresponding probabilities. A probability distribution can be in the form of a table, graph or mathematical formula.

Example

The table to the right shows the probability distribution for the random variable *X*, where *X* represents the number of movies streamed on Netflix each month.

x	$P(x)$
0	0.06
1	0.58
2	0.22
3	0.10
4	0.03
5	0.01

Rules for a Discrete Probability Distribution

Let $P(x)$ denote the probability that the random variable X equals x; then

1. $\Sigma P(x) = 1$
2. $0 \le P(x) \le 1$

Example

Which of these are a probability distribution?

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	0.01

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	– 0.01

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	0.04

Construct Probability Histograms

A **probability histogram** is a histogram in which the horizontal axis corresponds to the value of the random variable and the vertical axis represents the probability of that value of the random variable.

Example

Draw a probability histogram of the probability distribution to the right, which represents the number of movies streamed on Netflix each month

x	$P(x)$
0	0.06
1	0.58
2	0.22
3	0.10
4	0.03
5	0.01

The Mean of a Discrete Random Variable

The Mean of a Discrete Random Variable

The mean of a discrete random variable is given by the formula

$$\mu_X = \Sigma[x \cdot P(x)]$$

where x is the value of the random variable and $P(x)$ is the probability of observing the value x.

DVD Rental Example

Compute the mean of the probability distribution to the right, which represents the number of DVDs a person rents from a video store during a single visit.

x	***P*(*x*)**
0	0.06
1	0.58
2	0.22
3	0.10
4	0.03
5	0.01

Interpretation of the Mean of a Discrete Random Variable

Suppose an experiment is repeated n independent times and the value of the random variable X is recorded. As the number of repetitions of the experiment increases, the mean value of the n trials will approach μ_X, the mean of the random variable X. In other words, let x_1 be the value of the random variable X after the first experiment, x_2 be the value of the random variable X after the second experiment, and so on. Then

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

The difference between $\bar{x}$ and μ_X gets closer to 0 as n increases.

DVD Rental Example cont.

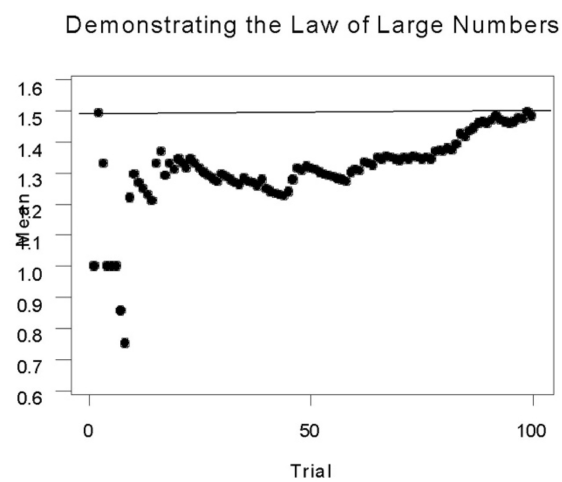
The following data represent the number of DVDs rented by 100 randomly selected customers in a single visit. Compute the mean number of DVDs rented.

1	1	1	1	1	1	1	2	2	2
2	1	1	1	1	1	3	1	1	3
1	1	2	1	1	1	1	2	3	0
0	1	1	1	1	1	1	1	4	1
1	3	1	2	2	1	3	1	1	1
1	2	1	1	3	1	1	2	3	2
0	0	1	1	3	1	2	1	2	3
0	2	1	1	1	1	1	3	3	1
5	1	1	2	2	3	1	2	2	4
2	2	2	0	1	2	1	1	1	0

$$\bar{X} = \frac{x_1 + x_2 + \ldots + x_{100}}{100} = 1.49$$

As the number of trials of the experiment increases, the mean number of rentals approaches the mean of the probability distribution.

Here is a graph of the mean after 1, then 2, then 3,… then 100 trials.



Computing and Interpreting the Mean of a Discrete Random Variable

Standard Deviation of a Discrete Random Variable

The standard deviation of a discrete random variable X is given by

$$\sigma_X = \sqrt{\sum\left[(x-\mu_x)^2 \cdot P(x)\right]}$$
$$= \sqrt{\sum\left[x^2 \cdot P(x)\right] - \mu_X^2}$$

where x is the value of the random variable, μ_X is the mean of the random variable, and $P(x)$ is the probability of observing a value of the random variable.

DVD Example Cont.

Compute the variance and standard deviation of the following probability distribution which represents the number of DVDs a person rents from a video store during a single visit.

x	$P(x)$
0	0.06
1	0.58
2	0.22
3	0.10
4	0.03
5	0.01