

1D: Average Rate of Change

Notes

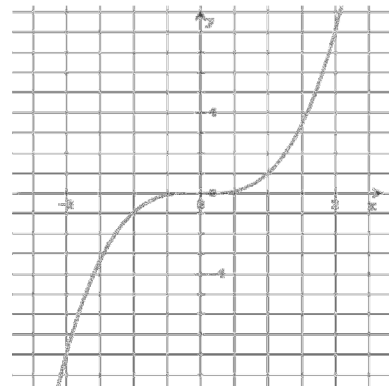
We have now learned how to describe the intervals in which functions increase or decrease, now we will consider *how much* they change in these intervals. We call this the **Average Rate of Change**.

The average rate of change for a function is the ratio of the change in the y value and the change in the x value over a given interval. So, the average rate of change for a function $f(x)$ on the interval $[a, b]$ is given by:

$$\text{Average Rate of Change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Consider This.

Consider the function $f(x) = x^3$ that is graphed to the right.



- Use the formula to find the average rate of change on the interval $[-1, 1]$.
- Draw a line through the points $(-1, -1)$ and $(1, 1)$.
- How does the line in (b) and the answer to (a) relate to other?

each

- Use the formula to find the average rate of change on the interval $[-1, 2]$.
- Show graphically and explain how you know that your answer in (d) is correct.

Key Correlation: The average rate of change of $f(x)$ on $[a, b]$ is the _____ of the secant line that passes through the points $(a, f(a))$ and $(b, f(b))$.

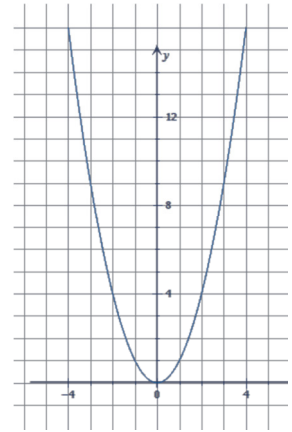
Assignment

1. Consider the function $f(x) = -2x + 3$.

- Use the formula to find the average rate of change for $f(x)$ on the interval $[2,6]$.
- Can you predict the average rate of change for $f(x)$ on the interval $\left[-\frac{3}{8}, \frac{5}{64}\right]$ without using the formula? Explain.

2. Find the rate of change for $g(x) = x^2$ over the following intervals

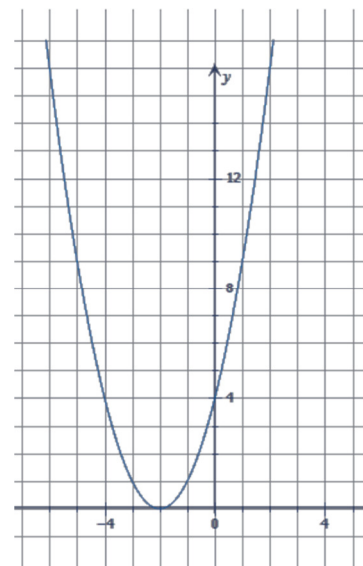
- $[0,1]$
- $[1,2]$
- $[2,3]$
- $[3,4]$



- Draw secant lines on the graph to the right of $y = x^2$ to verify your answers to parts (a)-(d).
- Use the formula to find the rate of change for $p(x) = x^2$ on $[x, x + h]$. Simplify your answer.

3. Find the average rate of change for $h(x) = (x + 2)^2$ on each of the following intervals.

- $[-2,2]$
- $[-1,1]$
- $[-1/2, 1/2]$
- Draw secant lines for each of these intervals.



4. Find the average rate of change for $j(x) = 2x^2 - 4$ on the following intervals. Justify your answer with work.

a. $[3,8]$

b. $[-10,-2]$

- c. One of the two intervals above should be positive and one is negative. Explain why.

- d. Use the formula to find the rate of change for $j(x)$ on $[x, x + h]$. Simplify your answer.

5. Consider the interval $[-5,1]$. Find the Average rate of change for the following functions.

a. $k(x) = x^3 + x$

b. $m(x) = x^3 + x + 3$

c. $n(x) = x^3 + x + 10$

- d. Does the value of the constant on the polynomials above change the Average Rate of Change. Explain.