4C: Problems with Logarithmic Functions

In this lesson we will use the properties of logarithms to solve equations that involve logarithms. We first need to recall these properties from earlier:

Properties of Logarithms

Let $b, R,$ and S be positive real numbers with $b \neq 1$, and c a

Product Rule: $\log_b(PQ) = \log_b P + \log_b Q$

Quotient Rule: $\log_b \frac{P}{Q} = \log_b P - \log_b Q$

Power Rule: $\log_b P^c = c \log_b P$

Where $b, R,$ and S be positive real numbers with $b \neq 1$, and c is any real number.

Example Assume that x and y are positive below.

a) Write as a sum of logarithms with no exponents: $\log \frac{3x^2}{y}$

b) Write as a single logarithm: $3 \ln 2 - 2 \ln 4 + \frac{1}{2} \ln 16$

To solve equations with logarithms, we can do one of the following:

1. Move all logarithms to one side, simplify to make one logarithm, then convert to exponential form and solve.
2. Write both sides of the equation as *one* logarithm with the same base, convert to exponential form, and solve.

After solving, you must check your domain to be sure that x is in the domain of the original function.

Example. Solve.

a) $\log_3(x + 1) = 4$

b) $2\log x + 3 \log 2 = \log 16$

Exercises

Assuming x and y are positive, use the properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms.

1. $\log_2 y^5$
2. $\log_2 \frac{2x^3}{y^2}$
3. $\log 1000x^4$

Assuming x, y, and z are positive, use properties of logarithms to write the expression as a single logarithm.

4. $\ln y - \ln 3$
5. $4\log y - \log z$
6. $3\ln 2 - 2\ln 4$

Find the exact solution algebraically, obtain a numerical approximation, and check it by substituting into the original equation.

7. $\log_4(1-x) = 1$
8. $3\ln(x-3) + 4 = 5$
9. $3 - \log(x+2) = 5$
10. $\frac{1}{2}\ln(x+3) - \ln x = 0$
11. $\log x - \frac{1}{2}\log(x+4) = 1$
12. $\ln(x-3) + \ln(x+4) = 3\ln 2$
13. $\log(x-2) + \log(x+5) = 2\log 3$
14. Determine whether a linear, logarithmic, exponential, power, or logistic regression equation is the best model for the data using your calculator. Let x= years after 1890

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Alaska's Population	63.6	64.4	55.0	59.2	72.5	128.6	226.2	302.6	401.9	550.0	626.9

15. In Chemistry, pH value is determined by the hydrogen ion concentration (H^+) relative to pure water. The function for finding pH is $p(H^+) = 7 + \log\left(\frac{1}{H^+}\right)$.
    a. State the domain of the function $p(H^+)$.
    b. For lemon juice, we know $H^+ \approx 100{,}000$. Find the pH of lemon Juice.
    c. If a solution of bleach has a pH value of $p(H^+) = 13.2$, find the value of H^+.