4A: Graphing Exponential Functions

In this lesson, we will generalize exponential functions and explore their graphs.

Definition

An **exponential function** is of the form

$$f(x) = a(b^x)$$

Where a is nonzero, b is positive, and $b \neq 1$. The constant a is the _initial value_ and b is the _base_.

Try These:

Which of the following are exponential functions?

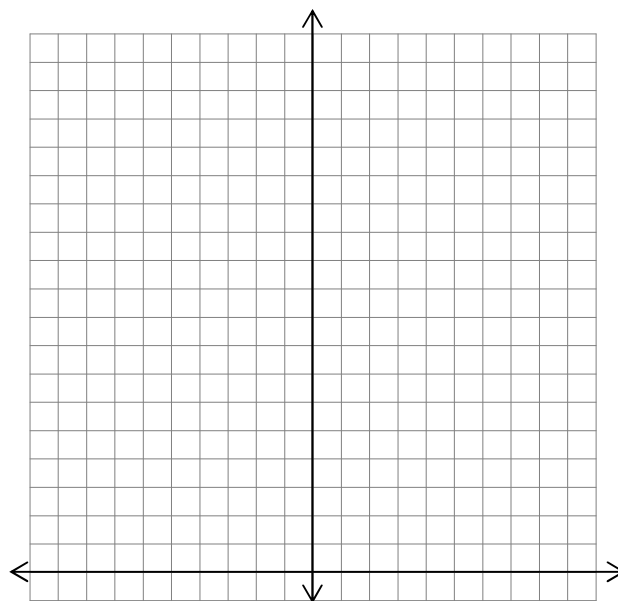a) $f(x) = 2.5^x$ b) $g(x) = 2(-3)^x$ c) $h(x) = .5(2^{-x})$ d) $j(x) = 2(x^3)$

Graph it

Graph the following functions by hand on the same axes:

$$y = 2^x \qquad\qquad y = \left(\tfrac{1}{2}\right)^x$$

Exploring Transformations

Now let's explore some transformations of exponential graphs. Use dynamic Algebra software (like Desmos), graph the following using sliders for the variable. Describe the change in the shape of the graph on the given interval.

- Graph the function $y = a(2^x)$ using a slider for a on $[-10,10]$
- Graph the function $y = b^x$ using a slider for b on $[-10, 10]$
- Graph the function $y = 2^{x+c}$ using a slider for c on $[-10,10]$
- Graph the function $y = 2^x + d$ using a slider fro d on $[-10,10]$

Transformations of Exponential Functions:

The values of a, b, c and d affect the graph of $y = a(b^{x+c}) + d$ as follows:

> a: Vertical stretch/shrink. If a is negative, it is reflected across the x-axis
> b: Rate of change.
>   >   If $b > 1$, the graph is **exponential growth** (increasing).
>   >   If $0 < b < 1$, the graph is **exponential decay** (decreasing).
> c: Horizontal shift.
>   >   *if c>0, graph moves left; if c<0, graph moves right.*
> d: Vertical shift.
>   >   *if d>0, graph moves up; If d<0, graph moves down.*

Below are several graphs of functions in the form $f(x) = ab^x$. With a partner, consider the following.

1. Which functions have positive a values, and which have negative a values?

2. Order the functions from largest a value to smallest a value. How did you decide this order?

3. Which functions have a b value that is greater than 1, and which functions have a b value that is less than 1?

4. Order the functions from largest b value to smallest a value. How did you decide this order?