## 3C Assignment: Graphing Advanced Rational Functions

Answer the following problems with as much detail, explanation, and work that is appropriate.

## 1. Consider the function:

$$y = \frac{2x^2}{x^2 - 9}$$

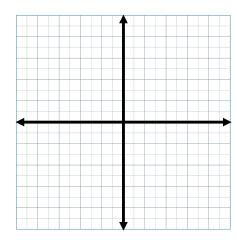
- a. Write the function in factored form and simplify it if possible.
- b. State the domain of the function.
- c. Write the equations of the vertical asymptote(x).

## d. Find these limits:

$$\lim_{x \to -3^{-}} \frac{2x^2}{x^2 - 9} = \lim_{x \to 3^{-}} \frac{2x^2}{x^2 - 9} =$$

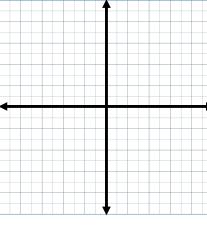
$$\lim_{x \to -3^+} \frac{2x^2}{x^2 - 9} = \lim_{x \to 3^+} \frac{2x^2}{x^2 - 9} =$$

- e. Divide the numerator and denominator  $x^2$  (the largest power of x in the denominator) and write the simplified result.
- f. Find the equation of the horizontal asymptote by considering the limit as  $x \to \infty$ .
- g. Graph the asymptotes (as doted lines) and the function here.

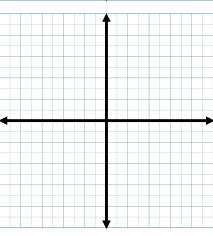


For 2 and 3 of the functions, list the equations of all the asymptotes, list the coordinates of all intercepts, then use the limits near the asymptotes to graph the function. Show all of these key features in an organized manner.

$$2. \quad b(x) = \frac{x}{x^2 - 16}$$



3. 
$$c(x) = \frac{x^2 - x - 6}{x^2 + 2x - 3}$$



4. Consider the function:

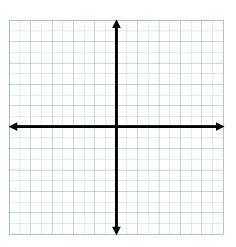
$$g(x) = \frac{x^2 + x}{2x - 4}$$

- a. Write the function in factored form and simplify it if possible.
- b. State the domain of the function.
- c. Write the equations of the vertical asymptote(x).
- d. Find these limits:

$$\lim_{x \to 2^{-}} \frac{x^2 + x}{2x - 4} = \lim_{x \to 2^{+}} \frac{x^2 + x}{2x - 4}$$

e. Divide the numerator and denominator of the original g(x) by x (the largest power of x in the denominator) and write the simplified result.

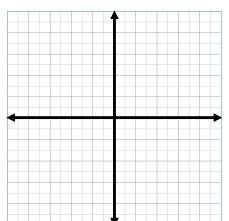
- f. Find the equation of the horizontal asymptote by considering the limit as  $x \to \infty$ .
- g. Graph the asymptotes (as doted lines) and the  $g(x) = \frac{x^2 + x}{2x 4}$  here.



For 5-7 of the functions, list the equations of all the asymptotes, list the coordinates of all intercepts, then use the limits near the asymptotes to graph the function. Show all of these key features in an organized manner.

5. 
$$d(x) = \frac{x^2 + 3x}{x + 2}$$

**6.** 
$$e(x) = \frac{-x^2 - x}{2x - 4}$$



7. Challenge Problem (optional)

$$f(x) = \frac{x^3}{x - 1}$$

