Pre-Calculus Name: Date: Period:

3A: Rational Functions and Asymptotes

Assignment

Find the key asymptotes and intercepts for each function, then graph them on the last page.

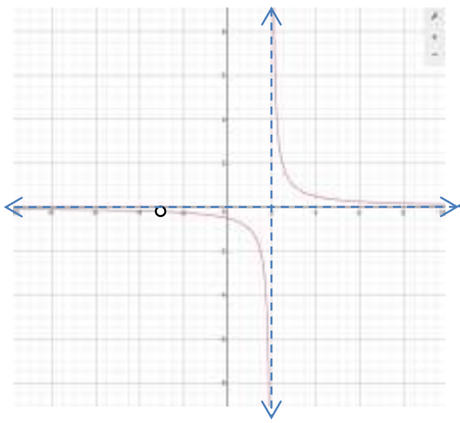
Function	Factored Form	**Domain Restrictions** (from Denom.) **and zeros** (from Numerator)	**Vertical Asymptotes (a)** and Removable Discontinuities	**Divide** by greatest power of x in denominator	**Limits** as $x \to \pm\infty$	**Horizontal Asymptotes** from $\lim_{x\to\infty} f(x)$
$a(x) = \frac{x+3}{x^2+x-6}$	$y = \frac{x+3}{(x+3)(x-2)}$ $y = \frac{1}{x-2}$	Domain Restrictions: $x \neq -3$ $x \neq 2$ Zeros: None	Vert. Asym.: $x = 2$ Hole: $x = -3$	$y = \frac{\frac{1}{x}}{\frac{x}{x} - \frac{2}{x}}$ $y = \frac{\frac{1}{x}}{1 - \frac{2}{x}}$	As $x \to -\infty$, $y \to 0$ As $x \to \infty$ $y \to 0$	$\lim_{x\to\pm\infty} f(x) = 0$ so Horizontal asymptote is $y = 0$

Function	Factored Form	**Domain Restrictions** (from Denom.) **and zeros** (from Numerator)	**Vertical Asymptotes** $(\boldsymbol{a})$ and Removable Discontinuities	**Divide** by greatest power of x in denominator	**Limits** as $x \to \pm\infty$	**Horizontal Asymptotes** from $\lim\limits_{x\to\infty} f(x)$
$b(x) = \dfrac{3}{4x-12}$	$y = \dfrac{3}{4(x-3)}$	Dom. Restrictions: $x \neq 3$ Zeros: None	Vert. Asym: $x = 3$ Holes: none	$y = \dfrac{\frac{3}{x}}{\frac{4x}{x} - \frac{12}{x}}$ $y = \dfrac{\frac{3}{x}}{4 - \frac{12}{x}}$	As $x \to -\infty$, $y \to 0$ As $x \to \infty$ $y \to 0$	$\lim\limits_{x\to\pm\infty} f(x) = 0$ so Horizontal asymptote is $y = 0$
$c(x) = \dfrac{x^2+x}{x^3-x}$	$y = \dfrac{x(x+1)}{x(x+1)(x-1)}$ $y = \dfrac{1}{x-1}$	Dom. Restrictions: $x \neq 0, 1, -1$ Zeros: None	Vert. Asym: $x = 1$ Holes: $x = 0, x = -1$	$y = \dfrac{\frac{1}{x}}{\frac{x}{x} - \frac{1}{x}}$ $y = \dfrac{\frac{1}{x}}{1 - \frac{1}{x}}$	As $x \to -\infty$, $y \to 0$ As $x \to \infty$ $y \to 0$	$\lim\limits_{x\to\pm\infty} f(x) = 0$ so Horizontal asymptote is $y = 0$
$d(x) = \dfrac{3x}{x+1}$	$y = \dfrac{3x}{x+1}$	Dom. Restrictions: $x \neq -1$ Zeros: $x = 0$	Vert. Asym: $x = 1$ Holes: None	$y = \dfrac{\frac{3x}{x}}{\frac{x}{x} + \frac{1}{x}}$ $y = \dfrac{3}{1 + \frac{1}{x}}$	As $x \to -\infty$, $y \to 3$ As $x \to \infty$ $y \to 3$	$\lim\limits_{x\to\pm\infty} f(x) = 3$ so Horizontal asymptote is $y = 3$

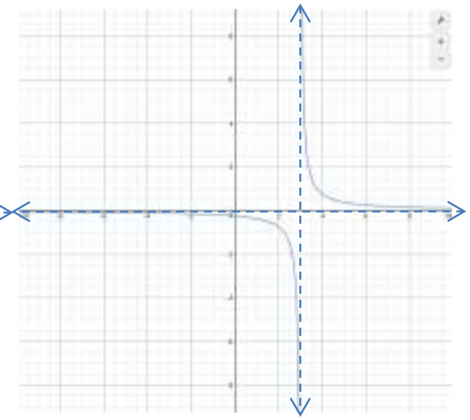
$e(x) = \frac{x^2 - 2x - 3}{x^2 + 4x + 3}$	$y = \frac{(x-2)(x+1)}{(x+1)(x+3)}$ $y = \frac{(x-2)}{(x+3)}$	Dom. Restrictions: $x \neq -1, 3$ Zeros: $x = 2$	Vert. Asym: $x = 3$ Holes: $x = -1$	$y = \frac{\left(\frac{x}{x} - \frac{2}{x}\right)}{\left(\frac{x}{x} + \frac{3}{x}\right)}$ $y = \frac{1 - \frac{2}{x}}{1 + \frac{3}{x}}$	As $x \to -\infty$, $y \to 1$ As $x \to \infty$ $y \to 1$	$\lim_{x \to \pm\infty} f(x) = 1$ so Horizontal asymptote is $y = 1$
$f(x) = \frac{2\,x^2 + x - 6}{x^2 + x - 2}$	$y = \frac{(x+2)(2x-3)}{(x+2)(x-1)}$ $y = \frac{(2x-3)}{(x-1)}$	$x \neq -2$ $x \neq 1$ *(by numerator: zero at* $x = \frac{3}{2}$*)*	Vert. Asym.: $x = 1$ Hole: $x = -2$	$y = \frac{\frac{2x}{x} - \frac{3}{x}}{\frac{x}{x} - \frac{1}{x}}$ $y = \frac{2 - \frac{3}{x}}{1 - \frac{1}{x}}$	As $x \to -\infty$, $y \to 2$ As $x \to \infty$ $y \to 2$	$\lim_{x \to \pm\infty} f(x) = 2$ so Horizontal asymptote is $y = 2$
$g(x) = \frac{x^2-4}{x^2+1}$	$\frac{(x+2)(x-2)}{x^2+1}$	*No Domain Restrictions*	*None because there are no domain restrictions*	$y = \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}}$ $y = \frac{1 - \frac{4}{x^2}}{1 + \frac{1}{x^2}}$	As $x \to -\infty$, $y \to 1$ As $x \to \infty$ $y \to 1$	From Limits: $y = 1$ *(by numerator: zero at* $x = \pm 2$*; and y-intercept at* $(0, -4)$*)*

Graphs

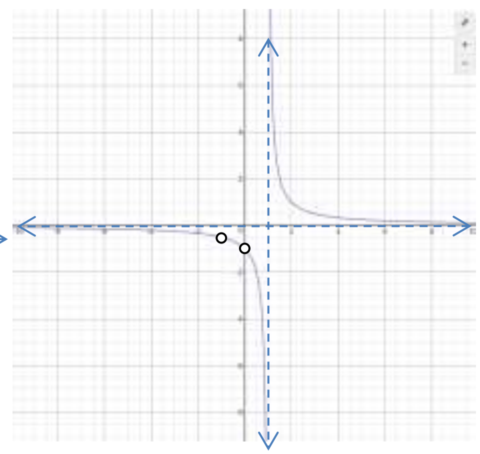
$a(x) = \frac{x+3}{x^2+x-6}$



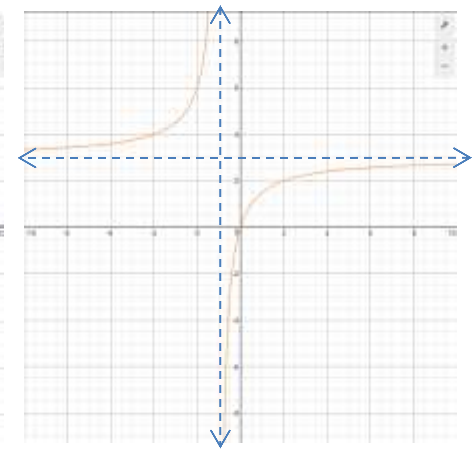
$b(x) = \frac{3}{4x-12}$



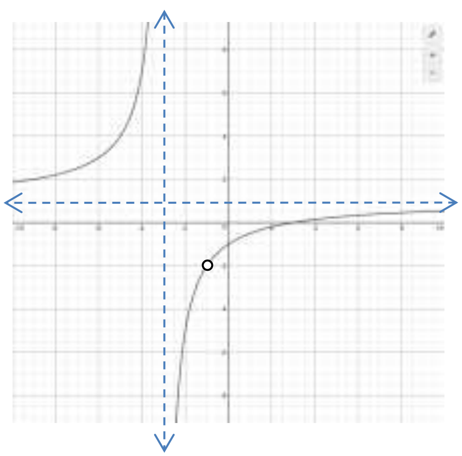
$c(x) = \frac{x^2+x}{x^3-x}$



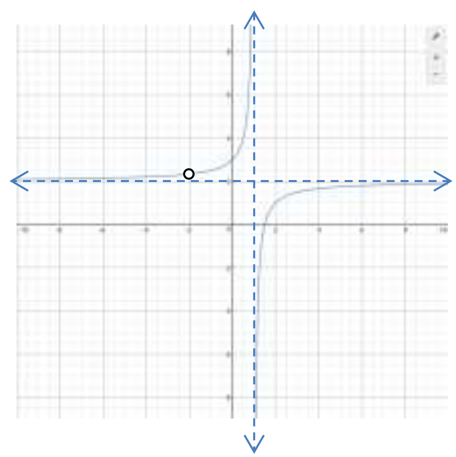
$d(x) = \frac{3x}{x+1}$



$e(x) = \frac{x^2-2x-3}{x^2+4x+3}$



$f(x) = \frac{2x^2+x-6}{x^2+x-2}$



$g(x) = \frac{x^2-4}{x^2+1}$