2C-2: Polynomial and Synthetic Division

Advanced Methods: Polynomial and Synthetic Division

When we cannot easily factor a polynomial function, then we must use more advanced techniques of polynomial and synthetic division.

Do you remember? Do you remember the standard algorithm for long-hand division? Use long-hand division to find $\frac{14567}{31}$.

Polynomial Division.

Simplify using polynomial division.

1. $\frac{3x^4-5x^3+6x^2-5x+1}{x-1}$

2. (This one will have a remainder! Don't forget to fill in any missing terms in the dividend) $\frac{x^4+2x^3+9x-1}{x^2+3x}$

Synthetic Division.

Simplify completely using synthetic division.

3. $\frac{3\,x^4-5\,x^3+6\,x^2-5\,x+1}{x-1}$

4. $\frac{3\,x^4-3\,x^3-17\,x^2-x-6}{(x-3)(x+2)}$

So, how do we know what factors we should try to factor out with synthetic division?

Rational root theorem:

If we have a function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0, \qquad a_n \neq 0$$

and $x = \frac{p}{q}$ is a rational root (a.k.a. "zero") of $f(x)$, where p and q are relatively prime, then p is a factor of a_0 and q is a factor of a_n .

In other words… ***only check for roots of the form*** $\boldsymbol{x = \frac{p}{q}}$ *that meet the conditions above.*

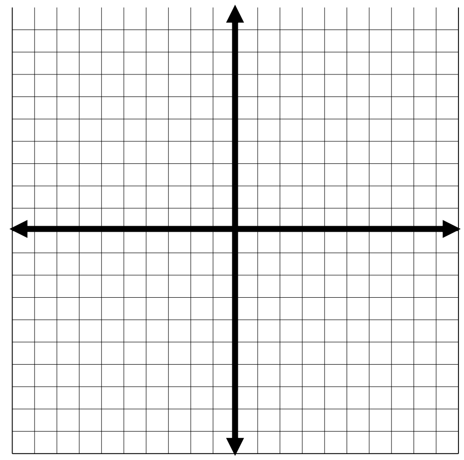
Example What are the *possible* rational roots of $f(x) = 2\,x^3 - 8\,x^2 - 18\,x + 72$?

Graphing Higher Degree Polynomials

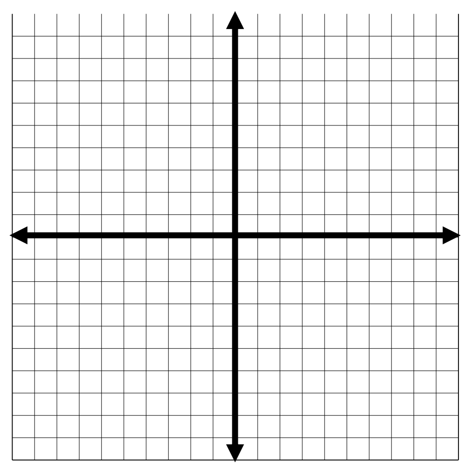
Try these:

Use the rational root theorem and synthetic division to completely factor and find all the *real* zeros of the functions below and graph

a) $f(x) = x^4 + 3\,x^3 - 6\,x - 4$



b) $y = 2\,x^4 - x^3 - 9\,x^2 + 4\,x + 4$



Equivalent statements:

1. The number k is a zero of the function f.
2. $x = k$ is a solution of $f(x) = 0$.
3. $(x - k)$ is a factor of $f(x)$.
4. k is an x-intercept of the graph of $f(x)$.