Pre-Calculus   Name: Date: Period:

2C: Power & Polynomial Functions

Power Functions:

Any function that can be written in the form

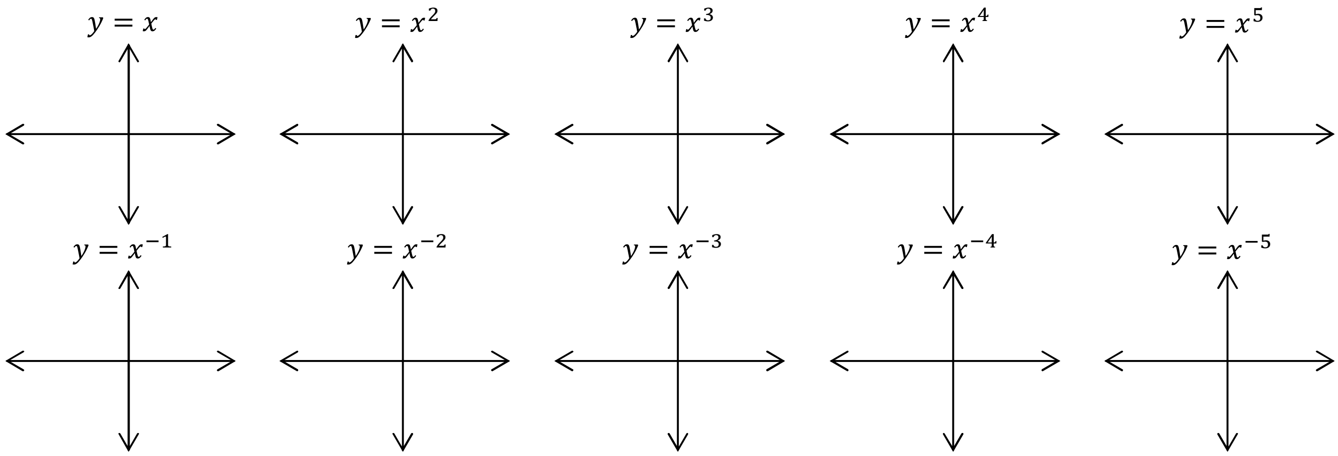
$$f(x) = a \cdot x^p$$

where a and p are nonzero constants, and p is the <u>power</u> and a is the <u>constant of variation.</u>

<u>Example:</u> In the power function $A = \pi r^2$, 2 is the power and π is the constant of variation.

Power Functions with $p \leq -1$ or $p \geq 1$

Use your calculator to sketch the graph of the following power functions on a window of $[-5,5] \times [-5,5]$



<u>Long Run Behavior:</u> *this describes how the value of the function changes for large values ($x \to \infty$) and large negative values ($x \to -\infty$). This is also known as the* ***end behavior.***

<u>Example:</u> Find the long run behavior of $f(x) = x^2$.

Solution: For the function $f(x) = x^2$ as $x \to -\infty$, $y \to \infty$; as $x \to \infty$, $y \to \infty$

<u>Conjectures:</u> Observe the graphs in your exploration above and complete the table with the continuity (is the graph continuous or discontinuous) and long-end behavior of each category of graph.

Value of p	Continuity?	Long-End Behavior	
Odd, $p \geq 1$		As $x \to -\infty$,	As $x \to \infty$,
Even, $p > 1$		As $x \to -\infty$,	As $x \to \infty$,
Odd, $p \leq -1$		As $x \to -\infty$,	As $x \to \infty$,
Even, $p < -1$		As $x \to -\infty$,	As $x \to \infty$,

<u>Consider this:</u> How does the value of a affect the graph of $f(x) = a\,x^p$?

Power Functions with $-1 < p < 1$

Example. Write the following in radical notation.

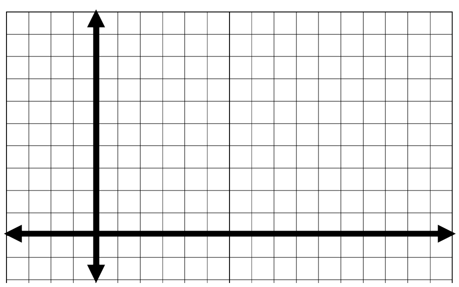
a) $x^{1/2}$

b) $x^{2/3}$

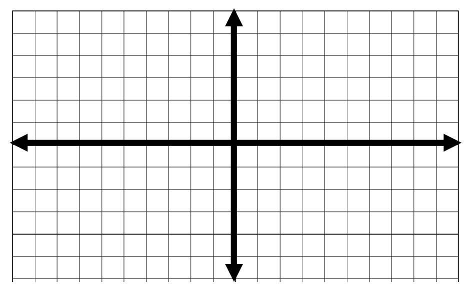
c) $x^{-2/5}$

Example Graph the following by hand

$y = x^{1/2}$

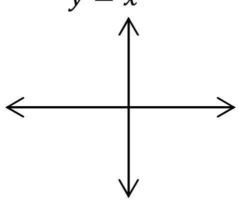


$y = x^{1/3}$

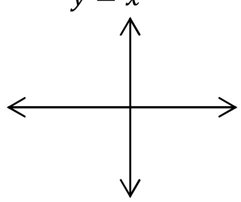


Use your calculator to sketch the graph of the following power functions on a window of $[-10,10] \times [-10,10]$

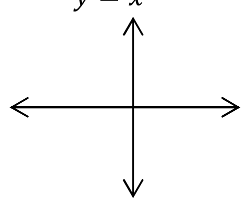
$y = x^{1/2}$



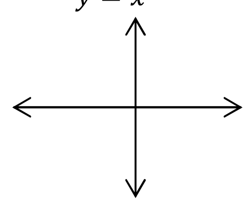
$y = x^{1/3}$



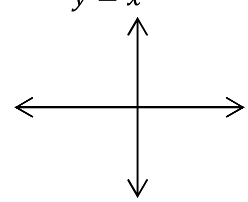
$y = x^{1/4}$



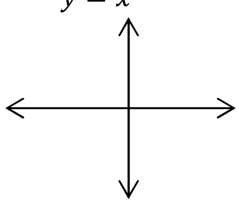
$y = x^{2/3}$



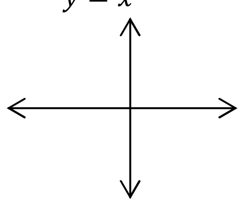
$y = x^{9/10}$



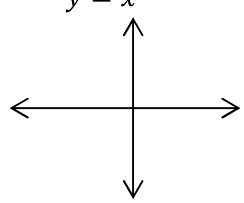
$y = x^{-1/2}$



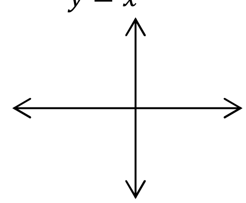
$y = x^{-1/3}$



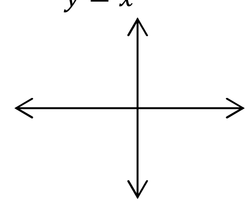
$y = x^{-1/4}$



$y = x^{-2/3}$



$y = x^{-9/10}$



Use your observations to complete these statements in as many ways a possible.

a) If $0 < a < 1$, then the graph of $y = x^a$ is

b) If $-1 < a < 0$, then the graph of $y = x^a$ is

Polynomials

A **Polynomial** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, \qquad a_n \neq 0$$

- Each monomial is called a ***term***
- The largest power n is called the ***degree***
- A polynomial with powers written in descending order is called ***standard form***
- The numbers $a_n, a_{n-1}, \ldots, a_0$ are called the ***coefficients*** of the polynomial
- The term $a_n x^n$ is called the ***leading term***, a_n is the ***leading coefficient***, and a_0 is the ***constant term***.
- If a polynomial has only one term, it is called a ***monomial.***

<u>Theorem:</u> *Polynomial Extrema and Zeros*

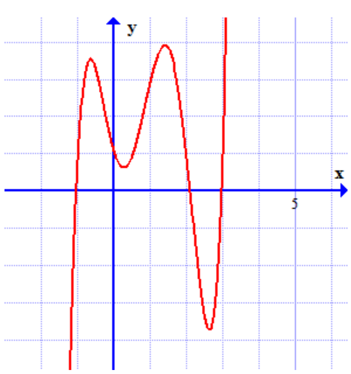
A polynomial function of degree n has at most $n - 1$ local extrema and at most n zeros.

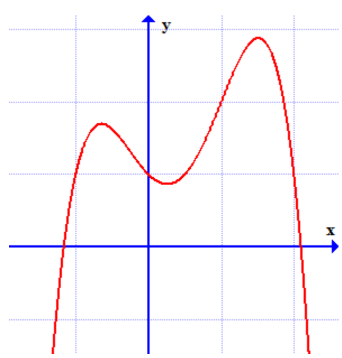
End Behavior and Intercepts

<u>Explore.</u> You explored the end behavior of basic power functions above, now use your graphing calculator to fill in the end behavior for the functions below.

	Leading Coefficient	**Degree**	**As $x \to \infty$, $y \to$**	**As $x \to -\infty$, $y \to$**	**# of x-intercepts**	**y-intercept**
$y = 2x^2 - 3x$						
$y = -2x^2 - 3x$						
$y = x^3 - 4x^2 + x + 2$						
$y = -2x^6 + 3x^5$						
$y = x^7 - 4x^4$						

<u>Example</u>: What is the least possible degree of the polynomial function in the graph shown?





<u>Example</u>: Find the vertical and horizontal intercepts of the function $f(x) = 4(x + 3)(x - 4)(x + 1)$.

Power Regression

Example

The table to the right shows the population of the United States from 1991 to 1998. Enter the data into a list in your calculator. Let x be the number of years after July 1, 1990.

Find a power regression function by choosing [PwrReg] from the [STAT]-[CALC] menu.

What is the function?

What is the value of R^2?

Use the function to predict the current population in the United States.

Date	National Population in millions
July 1, 1998	270.298
July 1, 1997	267.743
July 1, 1996	265.189
July 1, 1995	262.764
July 1, 1994	260.289
July 1, 1993	257.746
July 1, 1992	254.994
July 1, 1991	252.127