



Trigonometric Identities

Reciprocal Identities

$$\begin{aligned}\sin x &= \frac{1}{\csc x} & \cos x &= \frac{1}{\sec x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \sec x &= \frac{1}{\cos x} & \cot x &= \frac{1}{\tan x}\end{aligned}$$

Quotient Identities

$$\begin{aligned}\tan x &= \frac{\sin x}{\cos x} \\ \cot x &= \frac{\cos x}{\sin x}\end{aligned}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \cot^2 x = \csc^2 x \quad \tan^2 x + 1 = \sec^2 x$$

Co-Function Identities

$$\begin{aligned}\sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x\end{aligned}$$

Even-Odd Identities

$$\begin{aligned}\sin(-x) &= -\sin x & \cos(-x) &= \cos x & \tan(-x) &= -\tan x \\ \csc(-x) &= -\csc x & \sec(-x) &= \sec x & \cot(-x) &= -\cot x\end{aligned}$$

Angle-Sum (Difference) Identities

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

Double-Angle Identities

$$\begin{aligned}\sin(2u) &= 2 \sin u \cos u \\ \cos(2u) &= \begin{cases} \cos^2 u - \sin^2 u \\ 2 \cos^2 u - 1 \\ 1 - 2 \sin^2 u \end{cases} \\ \tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

Half-Angle Identities

$$\begin{aligned}\sin\left(\frac{u}{2}\right) &= \pm \sqrt{\frac{1 - \cos u}{2}} & \cos\left(\frac{u}{2}\right) &= \pm \sqrt{\frac{1 + \cos u}{2}} \\ \tan\left(\frac{u}{2}\right) &= \begin{cases} \pm \sqrt{\frac{1 - \cos u}{1 + \cos u}} \\ \frac{1 - \cos u}{\sin u} \\ \frac{\sin u}{1 + \cos u} \end{cases}\end{aligned}$$

Power-Reducing Identities

$$\begin{aligned}\sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u}\end{aligned}$$