9A.1: Trigonometric Identities - Examples

Using Identities to Simplify

These identities are useful when we need to replace one trigonometric ratio with a form of a different ratio.

Useful Steps:

1. Try writing functions in terms of sine and cosine.
2. Find connections between functions to try to cancel some values.
3. Our goal is to write the expression with as few trig. functions as possible and to avoid fractions in the simplified form.

Examples

Use the reciprocal and Pythagorean identities to rewrite and simplify the expressions as much as possible.

$$\boldsymbol{\csc x \cdot \tan x}$$

$$\begin{aligned} \csc x \cdot \tan x &= \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} \\ &= \frac{1}{\cos x} \\ &= \sec x \end{aligned}$$

$$\boldsymbol{\sin x \cdot \sec x \cdot \cot x}$$

$$\begin{aligned} \sin x \cdot \sec x \cdot \cot x &= \sin x \cdot \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} \\ &= \frac{\sin x \cdot \cos x}{\sin x \cdot \cos x} \\ &= 1 \end{aligned}$$

$$\boldsymbol{(\sin^2 x + \cos^2 x)(\cot^2 x + 1)}$$

$$\begin{aligned} (\sin^2 x + \cos^2 x)(\cot^2 x + 1) &= (1)(\csc^2 x) \\ &= \csc^2 x \end{aligned}$$

$$\boldsymbol{\frac{1 + \tan^2 x}{\csc^2 x}}$$

$$\begin{aligned} \frac{1 + \tan^2 x}{\csc^2 x} &= \frac{\sec^2 x}{\csc^2 \boldsymbol{x}} \\ &= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} \\ &= \frac{\sin^2 x}{\cos^2 x} \\ &= \tan^2 x \end{aligned}$$

$$(\mathbf{1}-\mathbf{cos^2}\,\boldsymbol{x})(\mathbf{csc^2}\,\boldsymbol{x}-\mathbf{1})$$

$$\begin{aligned}(1-\cos^2 x)(\csc^2 x-1) &= (\sin^2 x)(\cot^2 x)\\ &= \sin^2 x\cdot\frac{\cos^2 x}{\sin^2 x}\\ &= \cos^2 x\end{aligned}$$

$$\frac{\mathbf{sin^2}\,\boldsymbol{x}+\mathbf{cos^2}\,\boldsymbol{x}+\mathbf{tan^2}\,\boldsymbol{x}}{\mathbf{sec}\,\boldsymbol{x}}$$

$$\begin{aligned}\frac{\sin^2 x+\cos^2 x+\tan^2 x}{\sec x} &= \frac{1+\tan^2 x}{\sec \boldsymbol{x}}\\ &= \frac{\sec^2 x}{\sec x}\\ &= \sec x\end{aligned}$$

$$(\sec^2 x+\csc^2 x)-(\tan^2 x+\cot^2 x)$$

$You\ try\ this\ one!$

Useful Techniques

1. Direct Substitution (As we did above)
2. Change the identity to one involving only sines and cosines
3. Multiply or simplify parts of the expression to make a recognizable power of a trig. Function.
   Example:Simplify $(\sec x+1)(\sec x-1)\cdot\cot^2 x$
   (Note: Remember that $(a+b)(a-b)=a^2-b^2$, so we will multiply out the binomials first)

$$\begin{aligned}(\sec x+1)(\sec x-1)\cdot\cot^2 x &= (\sec^2 x-1)\cdot\cot^2 x\\ &= \tan^2 x\cot^2 x\\ &= \tan^2 x\ \cdot\frac{1}{\tan^2 x}\\ &= 1\end{aligned}$$

4. Factoring
   Example: *Simplify* $\cos^3 x+\cos x\sin^2 x$
   The key here is to look for a common factor.

$$\begin{aligned}\cos^3 x+\cos x\sin^2 x &= \cos x\,(\cos^2 x+\sin^2 x)\\ &= \cos x\cdot(1)\\ &= \cos x\end{aligned}$$

Practice Exercises

Simplify each expression using the trigonometric identities and substitution. Show your steps

1. $\cos x \cdot \tan x$
2. $\cos x \cdot \csc x$
3. $\csc x - \csc x \cdot \cos^2 x$
4. $\frac{1-\cos^2 x}{\sin x}$
5. $\frac{\sin^2 x+\tan^2 x+\cos^2 x}{\sec x}$
6. $\sin x + \sin x \tan^2 x$ *(Try factoring)*
7. $\frac{\sec^2 x-\tan^2 x}{\cos^2 x+\sin^2 x}$
8. *(Hint: Write in terms of sine and cosine. Then make a compound fraction)* $\frac{1+\tan x}{1+\cot x}$