Pre-Calculus Name: Date: Period:

9A/9B Review

Analytic Trigonometry: In advanced mathematics, the natural sciences, and engineering, it is sometimes necessary to simplify complicated trigonometric expressions and to solve equations that involve trigonometric functions.

Simplify

1. $\frac{\tan x}{\csc^2 x} + \frac{\tan x}{\sec^2 x}$

$$= \frac{\tan x}{\frac{1}{\sin^2 x}} + \frac{\tan x}{\frac{1}{\cos^2 x}}$$
$$= \tan x \sin^2 x + \tan x \cos^2 x$$
$$= \tan x \left(\sin^2 x + \cos^2 x\right)$$
$$= \tan x \cdot 1$$
$$= \tan x$$

2. $\frac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x}$

$$= \frac{\frac{1}{\cos^2 x} \cdot \frac{1}{\sin x}}{\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}}$$
$$= \frac{\frac{1}{\cos^2 x} \cdot \frac{1}{\sin x}}{\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}} \cdot \frac{\cos^2 x \sin^2 x}{\cos^2 x \sin^2 x}$$
$$= \frac{\sin x}{\sin^2 x + \cos^2 x}$$
$$= \frac{\sin x}{1}$$
$$= \sin x$$

Verify the following identities

3. $\sec\alpha - \cos\alpha = \sin\alpha \tan\alpha$

$$\sec\alpha - \cos\alpha = \frac{1}{\cos\alpha} - \cos\alpha$$
$$= \frac{1}{\cos\alpha} - \frac{\cos^2\alpha}{\cos\alpha}$$
$$= \frac{1 - \cos^2\alpha}{\cos\alpha}$$
$$= \frac{\sin^2\alpha}{\cos\alpha}$$
$$= \sin\alpha \cdot \frac{\sin\alpha}{\cos\alpha}$$
$$= \sin\alpha \tan\alpha$$

4. $\sec\theta = \sin\theta(\tan\theta + \cot\theta)$

$$\begin{aligned} sin\theta(\tan\theta + \cot\theta) &= \sin\theta\tan\theta + \sin\theta\cot\theta \\ &= \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \sin\theta \cdot \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta}{\cos\theta} + \cos\theta \\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} \\ &= \frac{1}{\cos\theta} \\ &= \sec\theta \end{aligned}$$

5. $\dfrac{\cos x}{1-\sin x} = \dfrac{1+\sin x}{\cos x}$

$$\begin{aligned} \frac{\cos x}{1-\sin x} &= \frac{1+\sin x}{1+\sin x} \cdot \frac{\cos x}{1-\sin x} \\ &= \frac{(1+\sin x)\cos x}{1-\sin^2 x} \\ &= \frac{(1+\sin x)\cos x}{\cos^2 x} \\ &= \frac{1+\sin x}{\cos x} \end{aligned}$$

6. $\dfrac{\sin t}{1-\cos t} = \csc t + \cot t$

$$\begin{aligned} \frac{\sin t}{1-\cos t} &= \frac{\sin t}{1-\cos t} \cdot \frac{1+\cos t}{1+\cos t} \\ &= \frac{\sin t\,(1+\cos t)}{1-\cos^2 t} \\ &= \frac{\sin t\,(1+\cos t)}{\sin^2 t} \\ &= \frac{1+\cos t}{\sin t} \\ &= \frac{1}{\sin t} + \frac{\cos t}{\sin t} \\ &= \csc t + \cot t \end{aligned}$$

7. $\sin u + \cos u \cot u = \csc u$

$$\sin u + \cos u \cot u = \sin u + \cos u \left(\frac{\cos u}{\sin u}\right)$$
$$= \frac{\sin^2 u}{\sin u} + \frac{\cos^2 u}{\sin u}$$
$$= \frac{\sin^2 u + \cos^2 u}{\sin u}$$
$$= \frac{1}{\sin u}$$
$$= \csc u$$

8. $\sin^2 x \cos^2 x + \cos^4 x = \cos^2 x$

$$\sin^2 x \cos^2 x + \cos^4 x = \cos^2 x (\sin^2 x + \cos^2 x)$$
$$= \cos^2 x \cdot 1$$
$$= \cos^2 x$$

9. $\dfrac{\cot(-\theta)}{\csc(-\theta)} = \cos\theta$

$$\frac{\cot(-\theta)}{\csc(-\theta)} = \frac{-\cot(\theta)}{-\csc(\theta)}$$
$$= \frac{\dfrac{\cos(\theta)}{\sin(\theta)}}{\dfrac{1}{\sin(\theta)}}$$
$$= \cos(\theta)$$

10. $\dfrac{2\sin^2 t + \sin t - 3}{1 - \cos^2 t - \sin t} = \dfrac{2\sin t + 3}{\sin t}$

$$\frac{2\sin^2 t + \sin t - 3}{1 - \cos^2 t - \sin t} = \frac{2\sin^2 t + \sin t - 3}{\sin^2 t - \sin t}$$
$$= \frac{(\sin t - 1)(2\sin t + 3)}{\sin t\,(\sin t - 1)}$$
$$= \frac{2\sin t + 3}{\sin t}$$

11. $\dfrac{\cos x}{1+\sin x}+\tan x=\sec x$

$$\begin{aligned}
\frac{\cos x}{1+\sin x}+\tan x &= \frac{1-\sin x}{1-\sin x}\cdot\frac{\cos x}{1+\sin x}+\tan x \\
&= \frac{\cos x\,(1-\sin x)}{1-\sin^2 x}+\tan x \\
&= \frac{\cos x\,(1-\sin x)}{\cos^2 x}+\tan x \\
&= \frac{(1-\sin x)}{\cos x}+\frac{\sin x}{\cos x} \\
&= \frac{(1-\sin x)+\sin x}{\cos x} \\
&= \frac{1}{\cos x} \\
&= \sec x
\end{aligned}$$