Unit 8 Toolkit – Graphs of Trigonometric Functions

This toolkit is a summary of some of the key topics you will need to master in this unit.

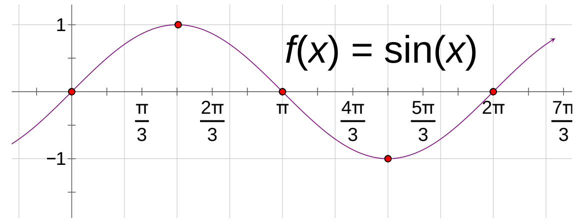
8A: Graphs of Sinusoids

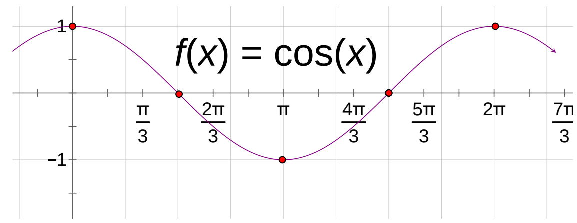
Learning Target: I can create and use graphs of transformations of sine and cosine functions to solve problems.

Characteristics of Sine and Cosine Curves
For **both** $f(x) = a\sin(bx + c) + d$ and $g(x) = a\cos(bx + c) + d$, we have

$$Amplitude = |a|, \qquad Period = \frac{2\pi}{|b|},$$

$$Frequency = \frac{|b|}{2\pi}$$





Steps for graphing transformations of sinusoids:

1. Graph the key points of the parent function with the same amplitude (a value) and period (b value)
   $y = a\sin(bx)$, *or* $y = a\cos(bx)$
2. Then use the translations in the original function to find the final points.

Transformations of sine and cosine curves
For the functions $f(x) = a\sin(b(x - h)) + k$ and $g(x) = a\cos\big(b(x - h)\big) + k$, the constants $a, b, h,$ and k do the following:

a: Determines $amplitude = |a|$; (Vertical Stretch and reflect across the x-axis)
b: Determines $period = \frac{2\pi}{|b|}$ and $frequency = \frac{|b|}{2\pi}$. (Horizontal stretch and reflect across the y-axis)
h: Determines a phase shift of h units left or right. (Horizontal translation)
k: Translates the graph up or down k units. (Vertical translation)

Modeling with Sine or Cosine Functions
From the problem, we first determine maximum (M) and minimum (m) values, and the period (p). Then calculate the parameters of the function:

1. Determine amplitude: $A = \frac{1}{2}(M - m)$.
2. Determine vertical shift: $C = \frac{1}{2}(M + m)$.
3. Determine horizontal stretch factor: $B = \frac{2\pi}{p}$
4. Choose the appropriate sinusoid and phase shift (T)
   $f(t) = A\cos(B(t - T)) + C, \qquad or\ f(t) = A\sin(B(t - T) + C$
   (If necessary, you may make A negative.)

8B: Graphs of the other Trigonometric Functions

Learning Target: I can create and use the graphs of transformations of non-sinusoid trig. functions (csc, sec, tan, cot) to solve problems

Transformations of tangent and cotangent curves

For the functions $f(x) = a\tan(b(x-h)) + k$ and $g(x) = a\cot\big(b(x-h)\big) + k$, the constants $a, b, h,$ and k do the following:

a: Vertical stretch. With $b = 1$, curves contain points $\left(\frac{\pi}{4} + \pi n, \pm a\right)$ and $\left(\frac{-\pi}{4} + \pi n, \pm a\right)$

b: Horizontal stretch. Period $= \frac{\pi}{|b|}$

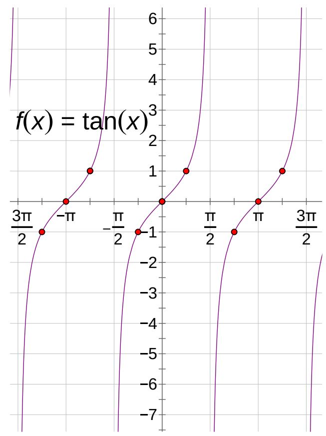
h: Horizontal translation. With $b = 1$,

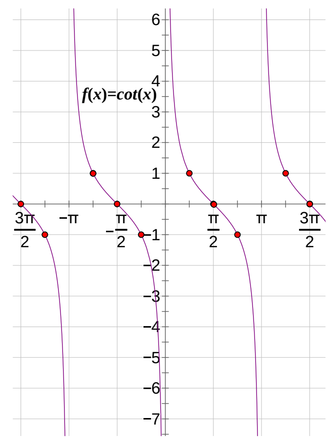
tangent zeros at $x = h + \pi n$, cotangent zeros at $x = \frac{\pi}{2} + h + \pi n$

k: Vertical translation.

Keys to graphing $f(x) = a\tan(b(x-h)) + k$ and $g(x) = a\cot\big(b(x-h)\big) + k$:

1. Begin by locating the asymptotes:
   a. Asymptotes of $y = \tan(bx)$ are at $x = \frac{\pi}{|2b|} + \frac{\pi}{|b|}n$ (shift this by h if $h \neq 0$)
   b. Asymptotes of $y = \cot(bx)$ are at $x = 0 + \frac{\pi}{|b|}n$ (shift this by h if $h \neq 0$)
2. Plot zeros half-way between all asyptotes on midline $y = k$
3. Plot the "quarter points" at $\frac{1}{4}$ and $\frac{3}{4}$ distance between asymptotes at heights of a and $-a$ Remember that Tangent is increasing, Cotangent is decreasing.
4. Draw curves





Transformations of Cosecant and Secant curves

For the functions $f(x) = a\csc(b(x-h)) + k$ and $g(x) = a\sec\big(b(x-h)\big) + k$, the constants $a, b, h,$ and k do the following:

a: Vertical stretch. The minimums and maximums are at $y = a$ and $y = -a$ when $k = 0$.

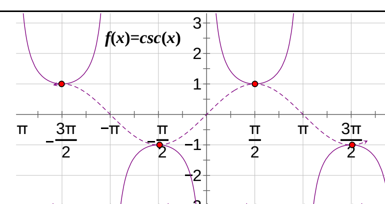
b: Horizontal Stretch. Period$=\frac{2\pi}{|b|}$.

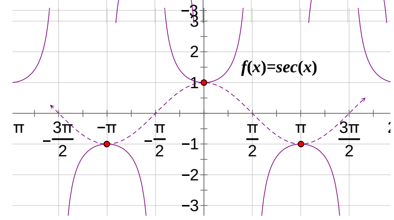
h: Horizontal shift.

k: Vertical shift.

Key to Graphing Cosecant and Secant:

1. To graph $f(x) = a\csc(b(x-h)) + k$, find the zeros, maximums, & minimums of $y = a\sin\big(b(x-h)\big) + k$
2. To graph $g(x) = a\sec\big(b(x-h)\big) + k$, find the zeros, maximums, & minimums of $y = a\cos\big(b(x-h)\big) + k$





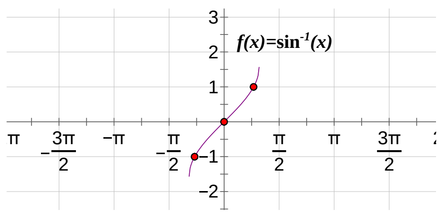
8C: Graphing Inverse Trigonometric Functions.

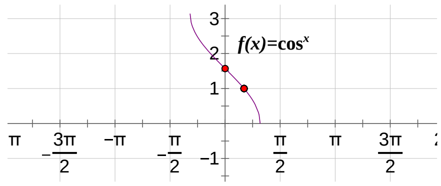
Learning Target: I can create and use graphs of transformations of inverse trigonometric functions to solve problems.

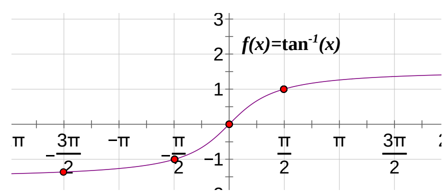
$$y = \sin^{-1} x, \quad or \;\; y = \arcsin x$$
$$y = \cos^{-1} x, \quad or \;\; y = \arccos x$$
$$y = \tan^{-1} x, \quad or \;\; y = \arctan x$$

Domain and Range

	$y = \sin x$	$y = \sin^{-1} x$	$y = \cos x$	$y = \cos^{-1} x$	$y = \tan x$	$y = \tan^{-1} x$
Domain	$(-\infty, \infty)$	$[-1,1]$	$(-\infty, \infty)$	$[-1,1]$	$x \neq \frac{\pi}{2} + \pi n$	$(-\infty, \infty)$
Range	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[-1,1]$	$[0, \pi]$	$(-\infty, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$







8D: Composite Graphs of Trigonometric Functions

Learning Target: I can create and use graphs of transformations of composite trigonometric functions to solve problems.

Sum of Functions:

Functions of the form $f(x) = \sin(x) + g(x)$ or $f(x) = \cos(x) + g(x)$ will oscillate about the function $g(x)$.

Sinusoids:

****Key: A sum (or difference) of two sinusoid functions is a sinusoid if they have the same period***

Damped Oscillation:

For a functions

$$f(x) = g(x) \sin x, \quad or \quad f(x) = g(x) \cos x$$

$g(x)$ is the *dampening factor.*

The graph will oscillate *between* $y = g(x)$ and $y = -g(x)$