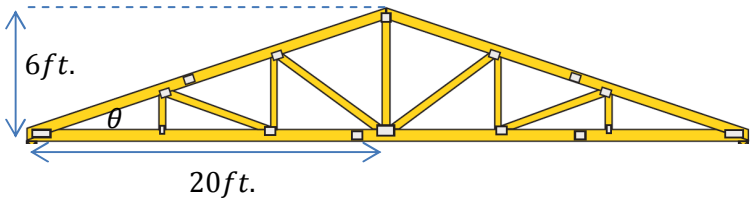
8C: Inverse Trig. Functions and Graphs

Angles from Ratios

Suppose we are building a roof truss like the one pictured and we want to figure out the angle to cut the boards. That is, we need to find the value of θ in the diagram.
Can you find the value of θ in degrees?



By definition, the trigonometric functions give us a ratio for a given angle. If we want to find an angle from a given ratio, we must use the "inverse" function. In the example above, we need to use the "inverse tangent" of $\frac{6}{20}$ which is written in the following ways:

$$\theta = \tan^{-1}\left(\frac{6}{20}\right), \quad or \quad \theta = \arctan\left(\frac{6}{20}\right), \quad or \quad \theta = \operatorname{atan}\left(\frac{6}{20}\right)$$

where the first two are the most standard ways.

When we read $\tan^{-1}\left(\frac{6}{20}\right)$, we interpret it as,

"*What is the measure of an angle that has a tangent of* $6/20$?"

Inverse Trigonometric Ratios as Functions

We now want to define inverse trigonometric ratios as functions. Keep in mind that when we have a function, we are required to maintain the characteristic that

"*for every x value in the domain, there is* ***only one*** *y value in the range.*"

Also, remember that the inverse of a function *reverses* the domain and range of the original function.

The inverse trig functions we will consider are

$$y = \sin^{-1} x, \quad or \quad y = \arcsin x$$
$$y = \cos^{-1} x, \quad or \quad y = \arccos x$$
$$y = \tan^{-1} x, \quad or \quad y = \arctan x$$

What's the Domain and Range?

Consider the definition of an inverse function and fill in the domains and ranges below

	$y = \sin x$	$y = \sin^{-1} x$	$y = \cos x$	$y = \cos^{-1} x$	$y = \tan x$	$y = \tan^{-1} x$
Domain						
Range						

Graphing Inverse Functions

Try These Evaluate on the appropriate range without your calculator. (Think of the unit circle)

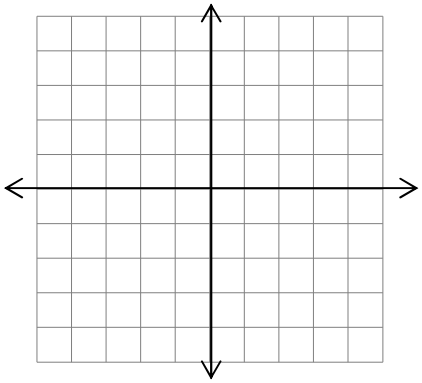
a. $\sin^{-1}(-1) =$

b. $\sin^{-1}\left(-\frac{1}{2}\right) =$

c. $\sin^{-1}(0) =$

d. $\sin^{-1}\left(-\frac{1}{2}\right) =$

e. $\sin^{-1}(1) =$



Graph It!

Now use the answers above to help you graph $y = \sin^{-1} x$ on the interval $[-1,1]$.

Try These Evaluate on the appropriate range without your calculator. (Think of the unit circle)

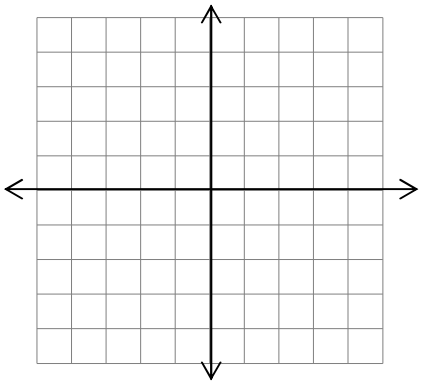
a. $\cos^{-1}(-1) =$

b. $\cos^{-1}\left(-\frac{1}{2}\right) =$

c. $\cos^{-1}(0) =$

d. $\cos^{-1}\left(-\frac{1}{2}\right) =$

e. $\cos^{-1}(1) =$



Graph It!

Now use the answers above to help you graph $y = \cos^{-1} x$ on the interval $[0, \pi]$.

Try These Evaluate on the appropriate range. Don't use your calculator for the first 3.

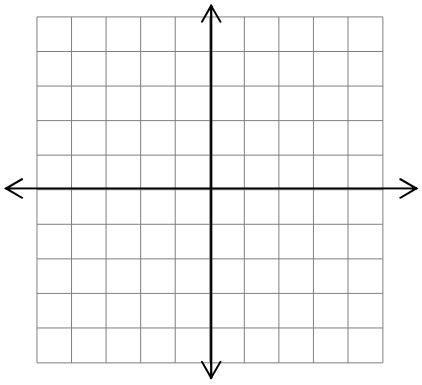
a. $\tan^{-1}(-1) =$

b. $\tan^{-1}(0) =$

c. $\tan^{-1}(1) =$

d. $\tan^{-1}(-2) =$

e. $\tan^{-1}(2) =$



Graph It!

Now use the answers above to help you graph $y = \tan^{-1} x$ on the interval $[-5,5]$.

Putting it all together

Find the exact value without using your calculator

1. $\sin^{-1}\left(\frac{1}{2}\right)$

2. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

3. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

4. $\tan^{-1}(1)$

5. $\sin\left(cos^{-1}\left(\frac{1}{2}\right)\right)$

6. $\arctan\left(\cos\left(\frac{\pi}{6}\right)\right)$

7. $\tan\left(\sin^{-1}\left(\frac{5}{7}\right)\right)$
   (Draw a reference Triangle)

8. $\tan(\arcsin(x))$
   (Draw a reference Triangle)

Assignment 8C: Inverse Trig Functions

State the range of the inverse function and find the value of the expression on that range. Support your answer by sketching a unit circle and the appropriate angle.

1. $\sin^{-1}\left(-\frac{1}{2}\right)$
2. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$
3. $\tan^{-1}(-\sqrt{3})$

Find the following values. Support your answer with a reference triangle or unit circle drawing.

1. $\arctan\left(\cos\left(\frac{\pi}{3}\right)\right)$
2. $\tan\left(\sin^{-1}\left(\frac{5}{6}\right)\right)$
3. $\sin(\arctan(x))$
   (State your answer in terms of x)
4. $\csc\left(\tan^{-1}\left(\frac{2}{x}\right)\right)$
   (State your answer in terms of x)
5. Matt and Brad are watching a rocket that they shot vertically into the air. They measure the angle of elevation, θ, from where the rocket was launched. If the launch pad is 100ft. away from the boys, write an equation that relates θ to the shortest distance, x, from the boys to the rocket.