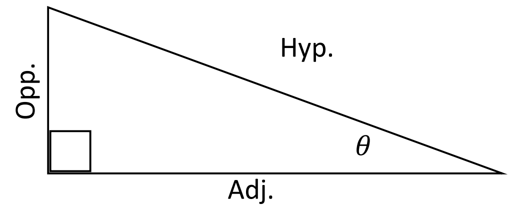
8B.2: Graphs of Cosecant and Secant

Or final two trigonometric functions to graph are cosecant and secant. Remember that
So, we predict that there is a close relationship between the graph of the sine and cosecant functions, and between the cosine and secant functions.

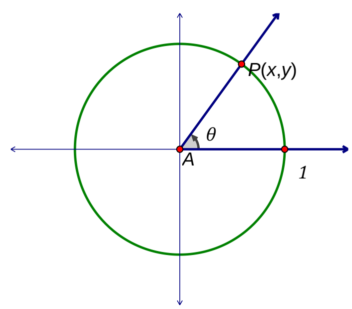


"Flipping" Sine Waves

Let's consider the relationship between values of the sine and cosecant functions.

Consider this:

a) Which values of θ give us $\sin\theta = \csc\theta$?



b) Is $\csc\theta$ defined for all values of θ?
If not, for which values of θ undefined?

What is the value of $\sin\theta$ at these values?

c) Explain why the sign (positive or negative) of $\sin\theta$ is the same as the sign of $\csc\theta$.
(Note: this is often written as $sgn(\sin\theta) = sgn(\csc\theta)$)

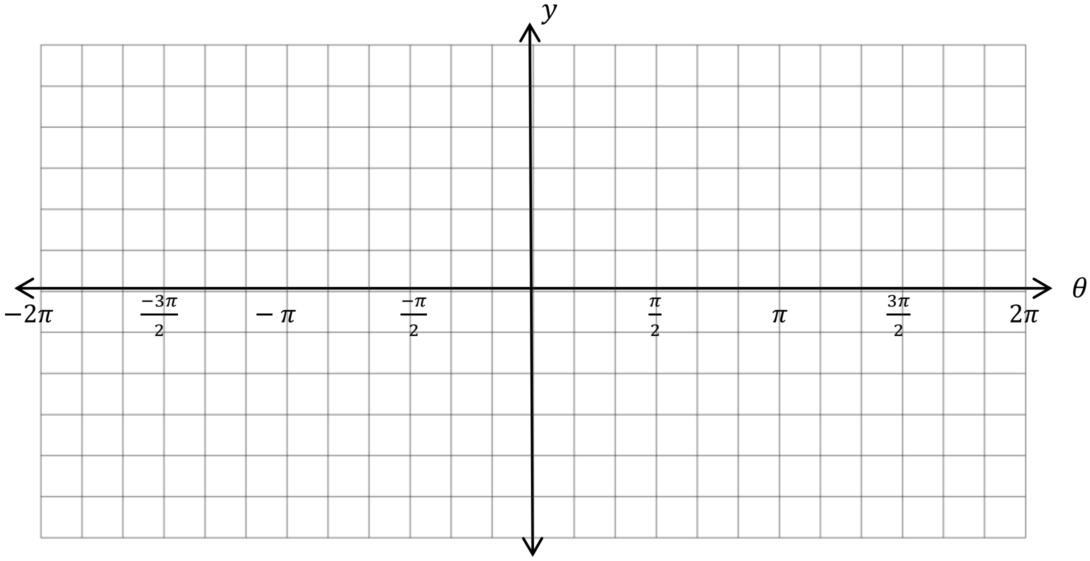
d) Complete the table for some key values of sine and cosecant

$\boldsymbol{\theta}$	$\boldsymbol{\sin\theta}$	$\boldsymbol{\csc\theta}$
0		
$\pi/6$		
$\pi/2$		
$5\pi/6$		
π		
$7\pi/6$		
$3\pi/2$		
$11\pi/6$		
2π		

Exploration: Graph by hand

Now we can use the values in the table above to give us a good picture of the graph of the cosecant function.

1. Plot the graph of $y = \sin\theta$ using a dotted curve.
2. Now draw in the vertical asymptotes for the graph of $y = \csc\theta$ (where cosecant is undefined).
3. Now plot each ordered pair $(\theta, \csc\theta)$ on the graph below for the angles on the interval $(0, 2\pi)$ and fill in the curve for $y = \csc\theta$
4. Finally, use the periodicity of the cosecant function to draw the graph of $y = \csc\theta$ on the interval $(-2\pi, 0)$

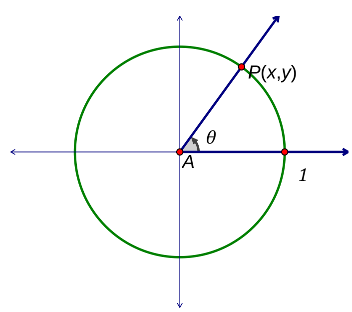


Some of the Same for Secant

Now let's consider the relationship between the cosine and secant functions in the same way.

Consider this:

a) Which values of θ give us $\cos\theta = \sec\theta$?



b) Is $\sec\theta$ defined for all values of θ?
If not, for which values of θ undefined?

What is the value of $\cos\theta$ at these values?

c) Explain why the sign (positive or negative) of $\sin\theta$ is the same as the sign of $\sec\theta$.
(Note: this is often written as $sgn(\cos\theta) = sgn(\sec\theta)$)

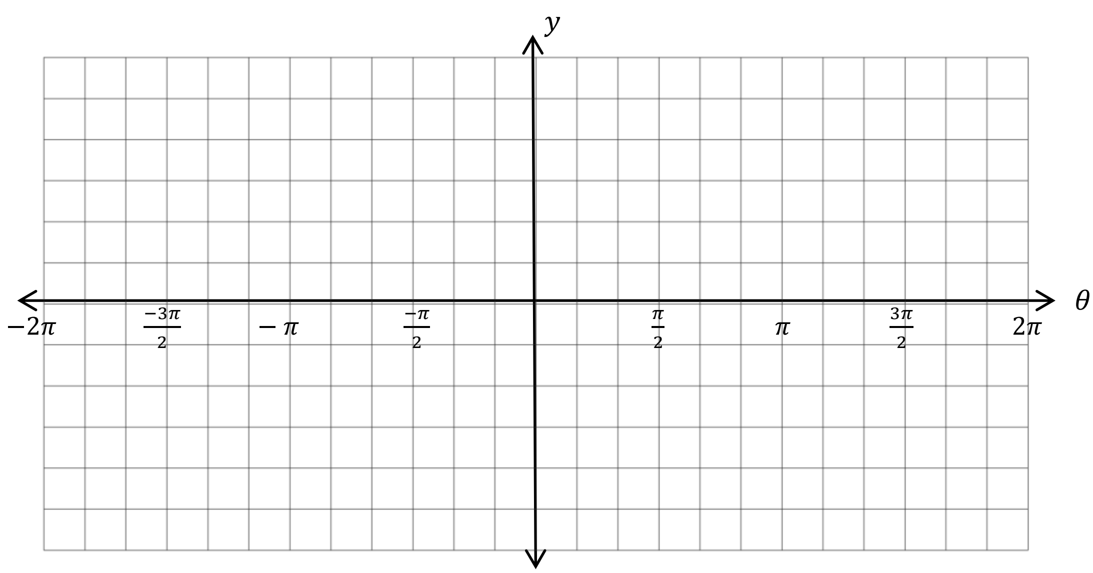
d) Complete the table for some key values of sine and cosecant

$\boldsymbol{\theta}$	$\boldsymbol{\cos\theta}$	$\boldsymbol{\sec\theta}$
0		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
π		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
2π		

Exploration: Graph by hand

Let's begin by exploring the values of the tangent function in the unit circle like we did with the sine and cosine function. Use the equations above to help.

5. Plot the graph of $y = \cos\theta$ using a dotted curve.
6. Now draw in the vertical asymptotes for the graph of $y = \sec\theta$ (where cosecant is undefined).
7. Now plot each ordered pair $(\theta, \sec\theta)$ on the graph below for the angles on the interval $(0, 2\pi)$ and fill in the curve for $y = \sec\theta$
8. Finally, use the periodicity of the cosecant function to draw the graph of $y = \sec\theta$ on the interval $(-2\pi, 0)$



Analyzing the Graphs

Use your graphs to find the following for $f(x) = \csc(x)$, and $y = \sec x$
(Describe periodic values as "any integer n")

	$y = \csc x$	$y = \sec x$
a. Domain:		
b. Range:		
c. Asymptotes		
d. Zeros		
e. Maximum or Minimum?		
f. Symmetry (odd or even)		
g. Period (horizontal distance required to repeat the curve):		

Transforming the Cosecant and Secant Curves

We now want to transform the graphs of $y = \csc x$ and $y = \sec x$. Use your graphing calculator to explore and determine how the constants $a, b, h,$ and k affect the graph of

$$f(x) = a\csc(b(x-h)) + k \text{ and } g(x) = a\sec\big(b(x-h)\big) + k$$

<u>Transformations of Cosecant and Secant curves</u>

For the functions $f(x) = a\csc(b(x-h)) + k$ and $g(x) = a\sec\big(b(x-h)\big) + k$, the constants $a, b, h,$ and k do the following:

a: Vertical stretch.

The minimums and maximums are at $y = a$ and $y = -a$ when $k = 0$.

b: Horizontal Stretch. Period$=\frac{2\pi}{|b|}$.

h: Horizontal shift.

k: Vertical shift.

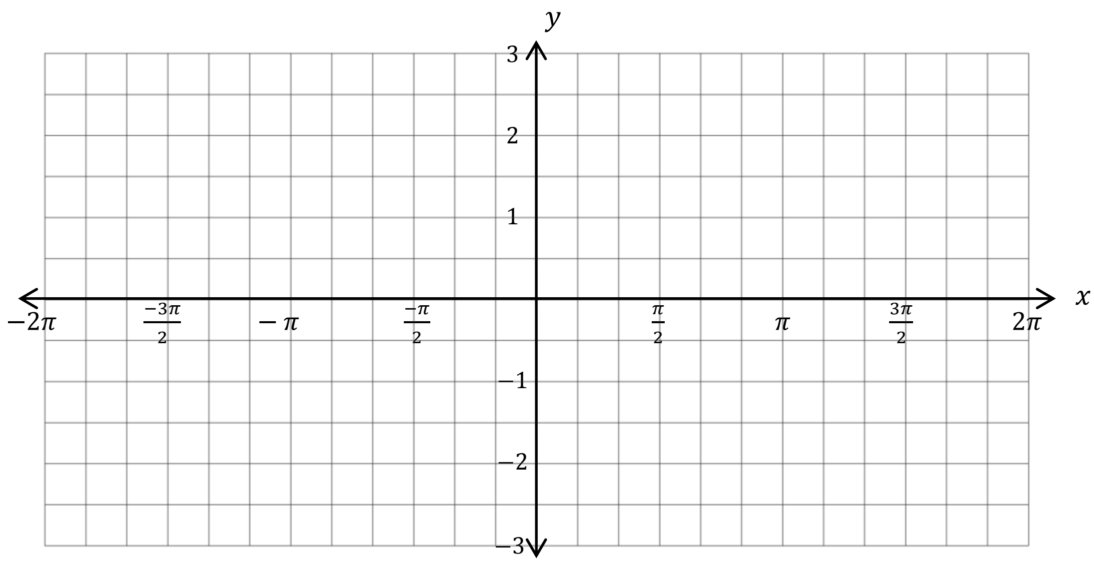
<u>Key to Graphing Cosecant and Secant:</u>

- To graph $f(x) = a\csc(b(x-h)) + k$, find the zeros, maximums, & minimums of $y = a\sin\big(b(x-h)\big) + k$

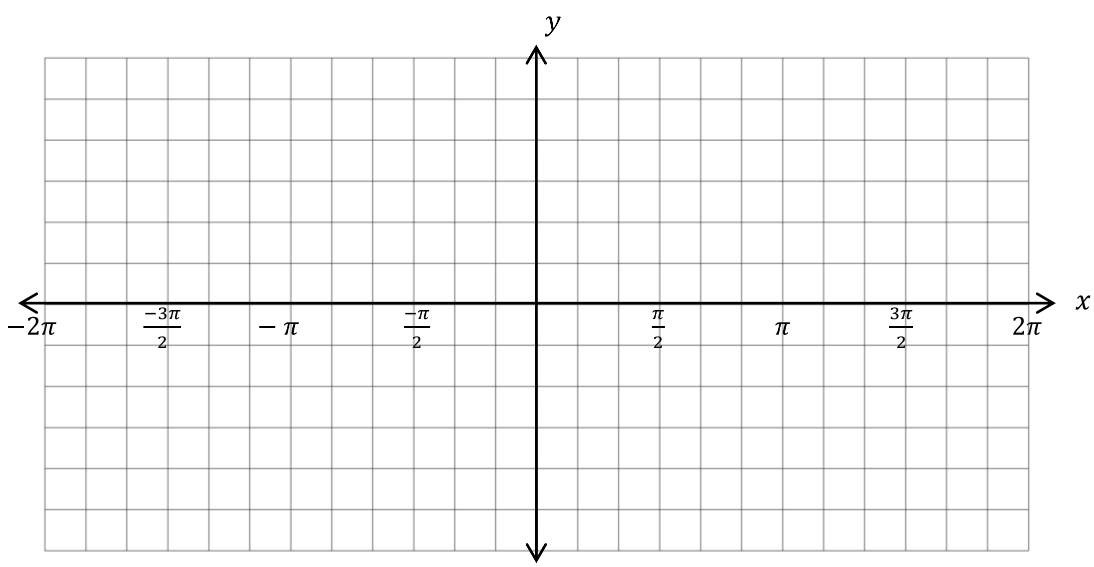
- To graph $g(x) = a\sec\big(b(x-h)\big) + k$, find the zeros, maximums, & minimums of $y = a\cos\big(b(x-h)\big) + k$

Try it! Determine the vertical stretch and period, and then use transformations to graph the following: Graph them without your calculator, then check them on your calculator

$y = 3 \csc x$



$y = \csc 2x - 1$



Pre-Calculus Name: Date: Period:

Assignment 8B.2: Cosecant and Secant Graphs

Describe how the graph the following curves differs from $y = \csc x$ *and* $y = \sec x$ and state the values of the asymptotes.

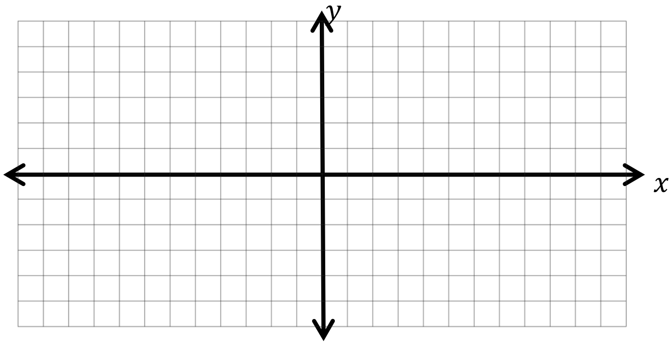
1. $y = 5\csc(x)$

2. $y = -\sec\left(\frac{x}{4}\right) + 3$

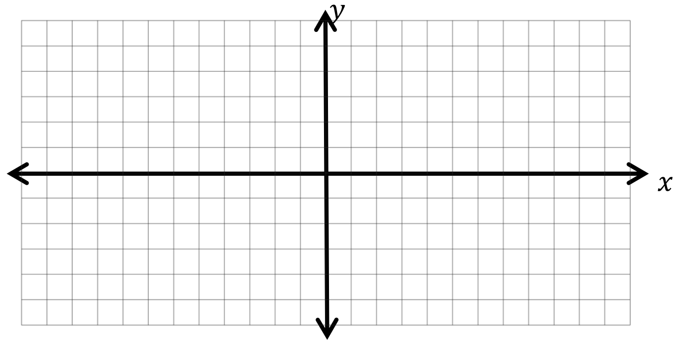
3. $y = \csc\left(2\left(x - \frac{\pi}{4}\right)\right) + 2$

Graph the following functions by first graphing their corresponding reciprocal function.

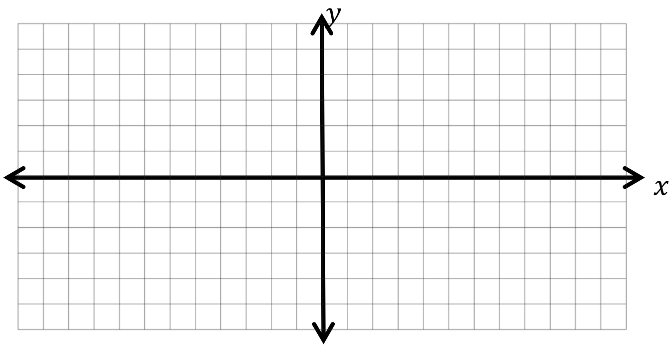
4. $y = 2\csc x$



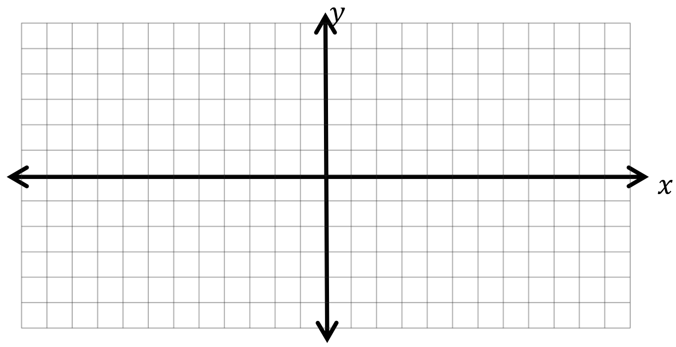
5. $y = \frac{1}{2}\csc\left(x + \frac{\pi}{4}\right)$



6. $y = \sec(2x)$



7. $y = \sec\left(x - \frac{\pi}{2}\right) + 1$



8. The "U" shapes in a secant or cosecant graph appear to be parabolas. Explain why the repeating shapes in the secant and cosecant graphs are *not* parabolas.

 Revised: 3/20/2017