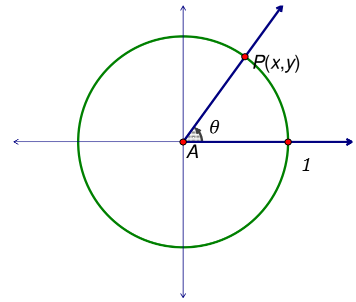
8A.2: Graphs of Sinusoids – Cosine Function



In the previous lesson, we graphed our first sinusoid function $y = a\sin(bx + c) + d$. We will now look at the cosine function curve and see how it's closely related to the sine function curve.

Exploration: Graph by hand

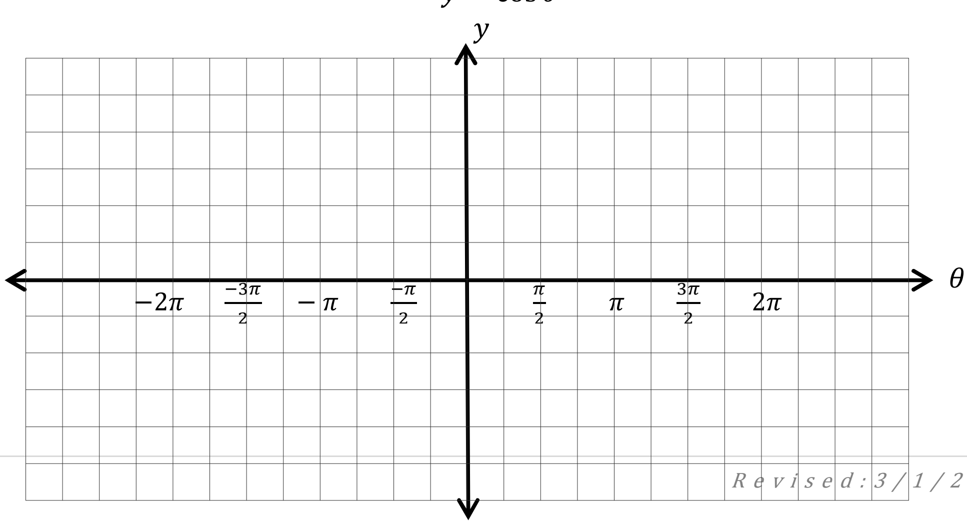
Let's begin by exploring the values of the cosine function in the unit circle like we did with the sine function. Remember that for the point $P(x, y)$ that is the intersection of the terminal side of central angle θ and the unit circle,
the x-coordinate is the value of $\cos\theta$.

1. Fill in the second column of the following table using our anchor angles in the unit circle (try to do this from memory as much as possible.)
   Notice, we are now finding cosines, not sines. Then use your calculator to approximate the decimal values using your π key and division (not using the cosine button.)

$\boldsymbol{\theta}$	$\boldsymbol{\cos\theta}$ (radical notation)	$\boldsymbol{\cos\theta}$ (to 4 decimal places)	Coterminal Angles
0			-2π
$\pi/4$			$-7\pi/4$
$\pi/2$			$-3\pi/2$
$3\pi/4$			$-5\pi/4$
π			$-\pi$
$5\pi/4$			$-3\pi/4$
$3\pi/2$			$-\pi/2$
$7\pi/4$			$-\pi/4$
2π			0

1. Now plot each ordered pair $(\theta, \cos\theta)$ on the graph below for the angles on the interval $(-2\pi, 2\pi)$.

$y = \cos\theta$



2. Use your graph to find the following for $f(x) = \cos(x)$
   a. Domain:
   b. Range:
   c. Local Maximum:
   d. Local Minimum:
   e. x values that give a maximum:
   f. x values that give a minimum:
   g. Symmetry (odd or even)
   h. Amplitude(half the height of the wave):
   i. Period (horizontal distance required to repeat the curve):

3. Did you notice that the cosine graph is very similar to the sine graph?
   Graph and label the function $y = \sin x$ on the same axes above using a different color pencil or pen or a dotted line.

4. Use what we learned about the transformations of the sine function to find the values of $a, b, c,$ and d such that the graphs of the following functions are identical:
   $$f(x) = a\sin(bx + c) + d, \qquad and \qquad g(x) = \cos(x)$$
   Graph these two on your calculator (in radian mode) to verify that they are the same.

Translating the Cosine Curve

In the last question, you should have found that the graphs of

$$f(x) = \sin\left(x + \frac{\pi}{2}\right)$$

and

$$g(x) = \cos(x)$$

are identical. When we shift a sine or cosine wave horizontally, we call this a **phase shift.**

Because we can write any cosine function as a sine function, they have similar features:

Characteristics of Sine and Cosine Curves
For **both** $f(x) = a\sin(bx + c) + d$ and $g(x) = a\cos(bx + c) + d$, we have
$$Amplitude = \lvert a \rvert, \qquad Period = \frac{2\pi}{\lvert b \rvert}, \qquad Frequency = \frac{\lvert b \rvert}{2\pi}$$

Maximums, Minimums, and Zeros

Use what you know about sine and cosine functions to fill in the table below to describe the locations for the maximums, minimums, and zeros for the sine and cosine functions

	$f(x) = \sin(x)$	$y = \cos(x)$
Location of Maximums		
Location of Minimums		
Location of Zeros		

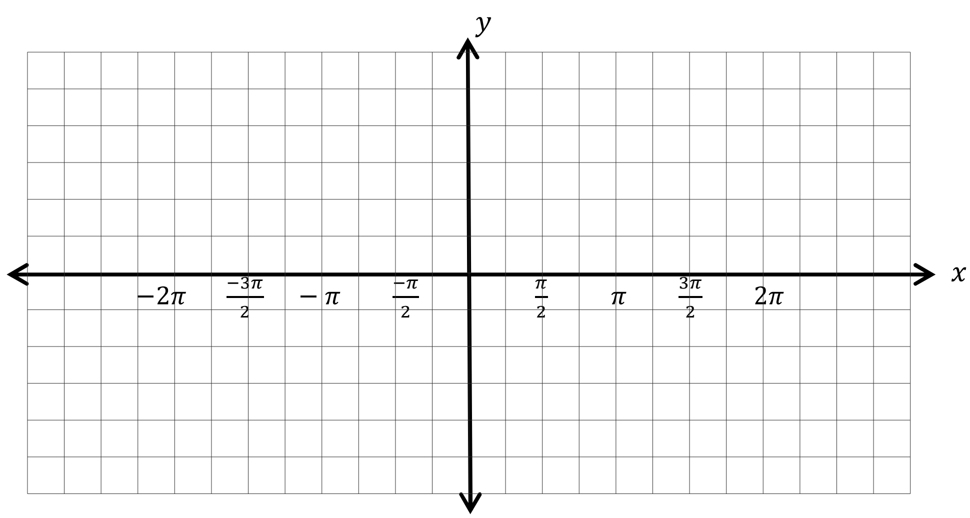
Since the cosine wave is a phase shift of a sine wave, they have the same transformations. In the last lesson, we considered functions of the form $f(x) = a\sin(bx + c) + d$. However, this form is less useful when we combine transformations. So, we will rewrite this in the form $f(x) = a\sin\big(b(x - h)\big) + k$

Transformations of sine and cosine curves
For the functions $f(x) = a\sin(b(x - h)) + k$ and $g(x) = a\cos\big(b(x - h)\big) + k$, the constants $a, b, h,$ and k do the following:

a: Determines $amplitude = |a|$
b: Determines $period = \frac{2\pi}{|b|}$ and $frequency = \frac{|b|}{2\pi}$.
h: Determines a phase shift of h units left or right.
k: Translates the graph up or down k units.

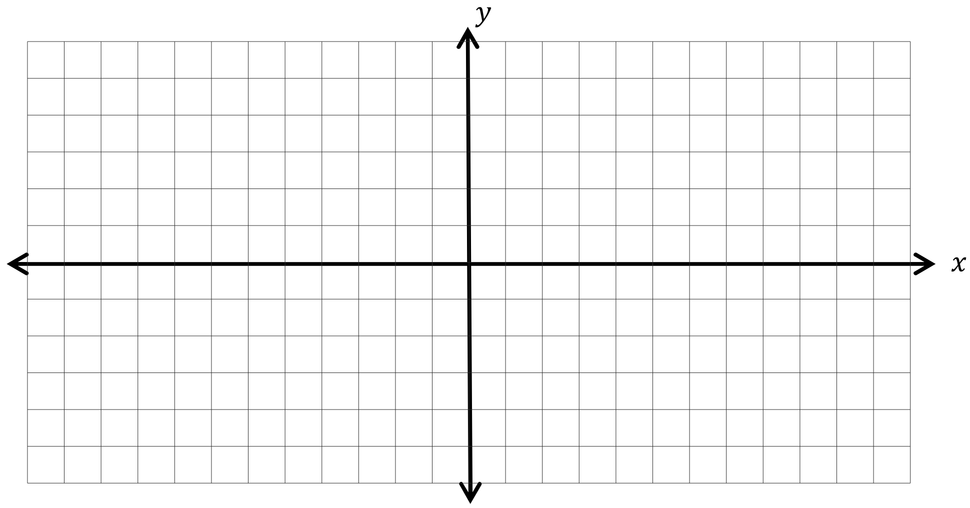
Try it! Determine the amplitude and period, and then use transformations to graph the following: Graph them without your calculator, then check them on your calculator.

a) $y = 3\cos x$

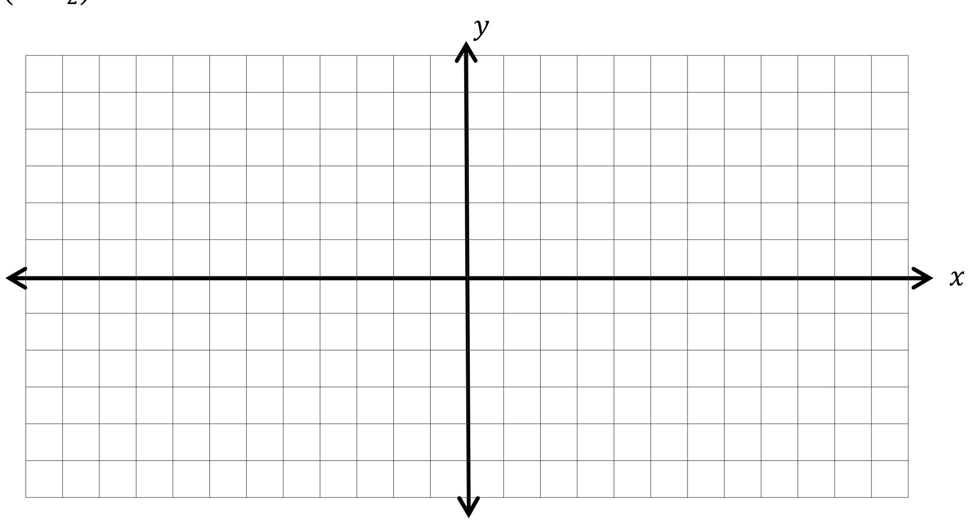


b) $y = \cos(2x)$

Note: To determine the scale for your x −axis, consider the your zeros, minimums, and maximums. Then set your scale to match these values on the grid-lines.



c) $y = \cos\left(x - \frac{\pi}{2}\right) + 1$



d) Write an equation for graph (c) using a sine function.