Pre-Calculus

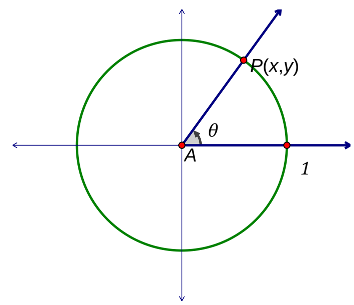
Name: Date: Period:

8A.1: Graphs of Sinusoids – Sine Function

We have seen that the six trigonometric functions are rooted in right triangles. We then expanded their application to any real numbered angle by defining the functions for any central angle on the unit circle. This application to all real numbers opens up the applications to an immense amount of possibilities such as radio waves, sound waves, spring oscillations, and light waves.

Exploration: Graph by hand

Let's begin by exploring the values of the sine function in the unit circle. Consider any point $P(x, y)$ that is the intersection of the terminal side of some central angle θ and the unit circle.

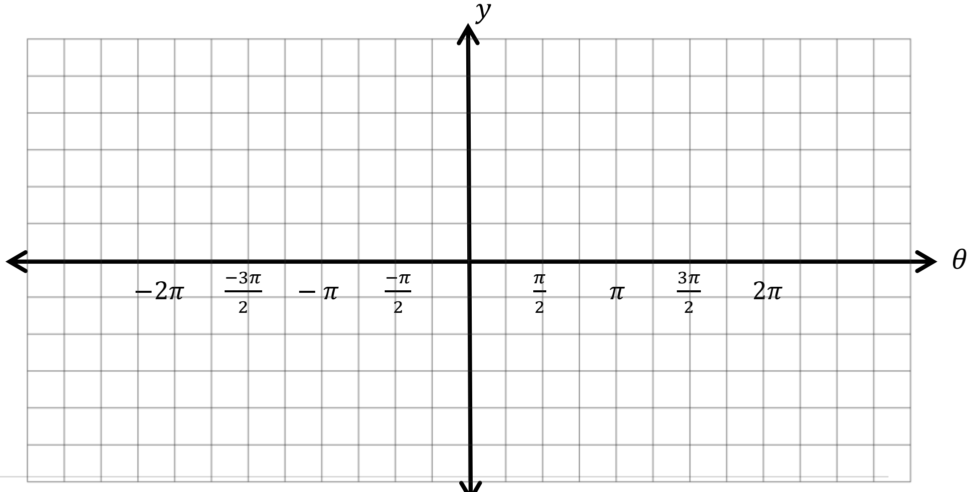


Which coordinate of P tells us the sine of θ?
Which coordinate of P tells us the cosine of ?

1. Fill in the following table using our anchor angles in the unit circle (try to do this from memory as much as possible.)

θ	$\sin\theta$ (radical notation)	$\sin\theta$ (to 4 decimal places)	Coterminal Angles (wait till step 3)
0			-2π
$\pi/4$			
$\pi/2$			
$3\pi/4$			
π			
$5\pi/4$			
$3\pi/2$			
$7\pi/4$			
2π			

2. Now plot each ordered pair $(\theta, \sin\theta)$ on the graph below for the angles on the interval $(0, 2\pi)$.
   $y = \sin\theta$



3. Now, let's consider negative angles. In the last column in the table, write the radian measure of the angles on the interval $(-2\pi, 0)$ that is coterminal to the angle in the first column. For example $-\frac{7\pi}{4}$ is coterminal to $\frac{\pi}{4}$, so we can write $-\frac{7\pi}{4}$ in the second row of the last column.

4. Now plot each ordered pair $(\theta, \sin\theta)$ on the graph below for the angles on the interval $(-2\pi, 0,)$ using the negative angles in the last column. Finish the graph by drawing in the curve through the points. Check your graph by setting your calculator to radian mode and graphing $y = \sin x$.

5. Use your graph to find the following for $f(x) = \sin(x)$
   a. Domain:
   b. Range:
   c. Local Maximum:
   d. Local Minimum:
   e. x values that give a maximum:
   f. x values that give a minimum:
   g. Symmetry (odd or even):
   h. Amplitude(half the height of the wave):
   i. Period (horizontal distance required to repeat the curve):

Exploration: Graph with Calculator

Now use a graphing calculator to change the curve:

$$y = a\sin\big(b(x + c)\big) + d$$

a. Choose different values for a and graph $y = a\sin x$
   Describe the changes that a makes to the graph:

b. Choose different integers for b and graph $y = \sin(bx)$
   Describe the changes that b makes

c. Choose different values for c and graph $y = \sin(x + c)$
   Describe the changes that c makes

d. Choose different values for d and graph $y = \sin x + d$
   Describe the changes that d makes

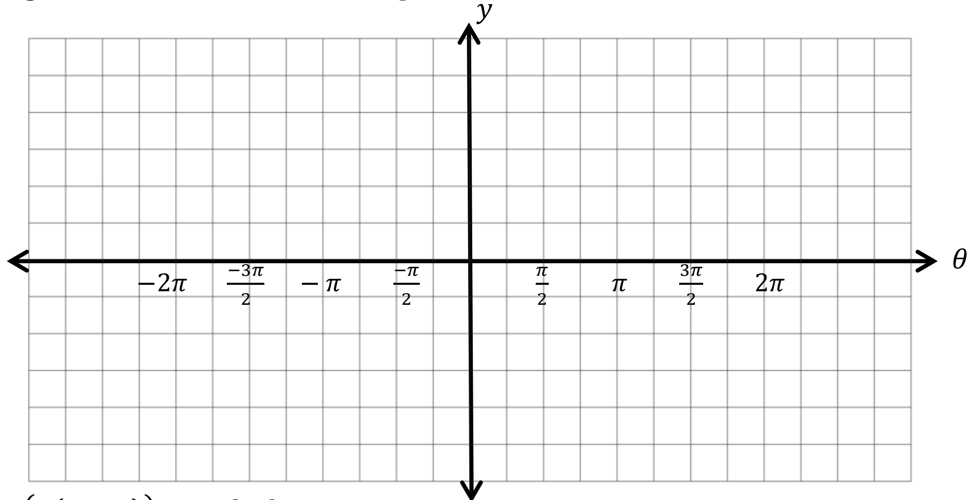
Key Features of a Sine wave:

- ***Amplitude:*** *half of the* *vertical distance* *from minimum to maximum (this is the* ***a*** *value)*
- ***Frequency:*** *The number of cycles per* 2π *(this is the* ***b*** *value)*
- ***Period:*** *The* *horizontal distance* *(a.k.a. time) needed for one complete cycle (this is* $\frac{2\pi}{b}$ *for sine.)*

Steps for graphing transformations of sinusoids:

1. Graph the key points of the parent function with the same amplitude (a value) and period (b value)
   $y = a\sin(bx)$, *or* $y = a\cos(bx)$
2. Then use the translations in the original function to find the final points.

Example Graph $y = 2\sin(x + \frac{\pi}{2})$



Example: Given the equation $y = 3\sin\big(6(x-2)\big) + 4$, find

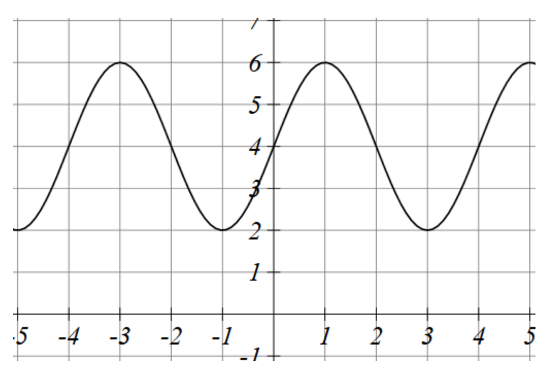
Amplitude=

Period=

Horizontal phase Shift=

Vertical shift (i.e. midline)=

Example: Use the graph to find the following of the sinusoid.



Amplitude=

Period =

Vetical shift (i.e. midline):

Equation:

Example: Find the period and horizontal phase shift of $y = 3\sin(5x - 30) + 4$