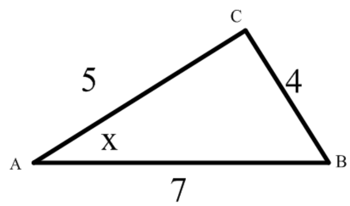
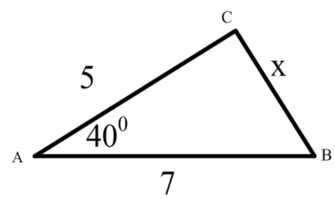
Pre-Calculus Name: **SOLUTIONS** Date: Period:

7D.2: The Law of Cosines

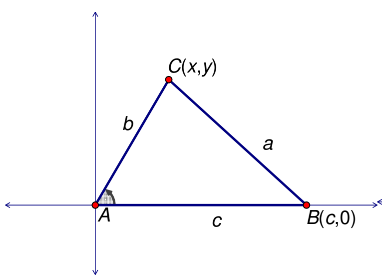
Consider this:

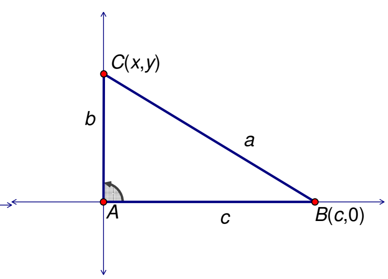
Can we use the Law of Sines to solve for x in these triangles? Explain.

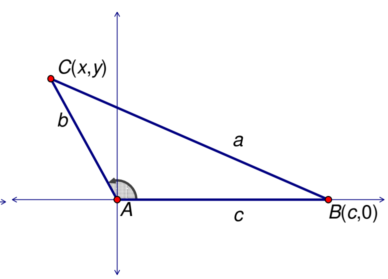




Suppose we make a triangle with a vertex at the origin, choose B on the x-axis, and choose point C above the x-axis. There are three possible types of triangles we can get.







Suppose we know the values of b, c, and $m\angle A$. Can we use the law of sines to solve for a? Since this will not work, we need a new method.

By Definition, we know

$$\cos A = \frac{x}{b}, \qquad \sin A = \frac{y}{b}$$

This gives us

$$x = b\cos A \qquad , \qquad y = b\sin A$$

Using the distance formula to describe the length of a, we get

$$a = \sqrt{(x-c)^2 + (y-0)^2}$$

Now, let's substitute and simplify…

$$a^2 = (b\cos A - c)^2 + (b\sin A)^2$$
$$a^2 = b^2(\cos A)^2 - 2bc(\cos A) + c^2 + b^2(\sin A)^2$$
$$a^2 = b^2\cos^2 A + b^2\sin^2 A + c^2 - 2bc(\cos A)$$
$$a^2 = b^2(\cos^2 A + \sin^2 A) + c^2 - 2bc(\cos A)$$

The *Pythagorean identity* tells us $\cos^2 A + \sin^2 A = 1$, so we simplify this expression to get

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

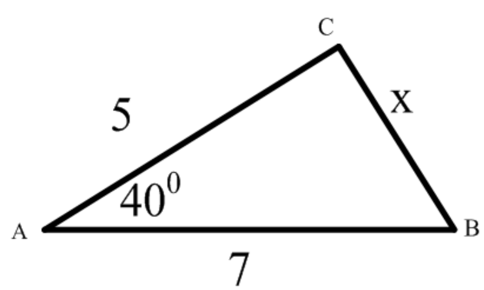
<u>Law of Cosines</u> If a triangle has angles $A, B,$ and C with opposite sides of length $a, b,$ and c, then the following must be true.

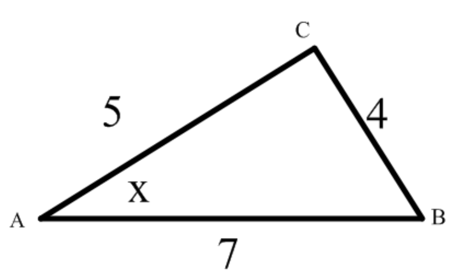
$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$
$$\boldsymbol{b^2 = a^2 + c^2 - 2ac\cos B}$$
$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos C}$$

<u>When can we use the Law of Cosines?</u> When you know two sides and the included angle (**S.A.S.**) or when you know three sides (**S.S.S.**), use the law of cosines to solve the triangle.

<u>Example 1</u>

Use the law of Cosines to find the value of x in the triangles below. Then solve the triangle.





<u>Example 2</u>

Use the law of cosines to solve the triangle.

$$a = 11, \qquad b = 35, \qquad b = 15$$

Exercises

1. Solve the following triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

a. ΔABC, if $A = 42^{\circ}, b = 12, c = 19$

$$a^2 = 12^2 + 19^2 - 2(12)(19)\cos 42$$
$$a = \sqrt{505 - 456\cos(42)}$$
$$a \approx 12.8890 \approx 12.9$$

$$12^2 = 12.8890^2 + 19^2 - 2(12.8890)(19)\cos B$$
$$B \approx 38.534^{\circ} \approx 38^{\circ}$$

$$42 + 38 + C = 180$$
$$C = 100^{\circ}$$

b. ΔPQR, if $P = 73^{\circ}, q = 7, r = 15$

$$p^2 = 7^2 + 15^2 - 2(7)(15)\cos 73$$
$$p = \sqrt{274 - 210\cos(73)}$$
$$p \approx 14.5809 \approx 14.6$$

$$7^2 = 14.5809^2 + 15^2 - 2(14.5809)(15)\cos Q$$
$$Q = 27.329 \approx 27^{\circ}$$

$$73 + 27 + R = 180$$
$$R \approx 80^{\circ}$$

c. ΔRST, if $r = 35, s = 22, and\ t = 25$

$$35^2 = 22^2 + 25^2 - 2(22)(25)\cos R$$
$$R = \cos^{-1}\left(\frac{-29}{275}\right) \approx 96.053 \approx 96^{\circ}$$

$$22^2 = 35^2 + 22^2 - 2(35)(22)\cos S$$
$$S = \cos^{-1}\left(\frac{35}{44}\right) \approx 37.302 \approx 37^{\circ}$$
$$96 + 37 + T = 180$$
$$T = 47^{\circ}$$

d. ΔBCD, if $B = 16^{\circ}, c = 27,\ d = 3$

$$b^2 = 27^2 + 3^2 - 2(27)(3)\cos 16$$
$$b = \sqrt{738 - 162co\,s(16)}\ or\ b = -\sqrt{738 - 162co\,s(16)}$$
$$b = \sqrt{738 - 162cos(16)} \approx 24.1304 \approx 24.1$$
$$27^2 = 3^2 + 24.1304^2 - 2(3)(24.1304)\cos C$$
$$27^2 = 3^2 + 24.1304^2 - 2(3)(24.1304)\cos C$$
$$C \approx 162.035$$

2. During her shift, a pilot flies from Columbus to Atlanta, a distance of 448 miles, and then on to Phoenix, a distance of 1583 miles. From Phoenix, she returns home to Columbus, a distance of 1667 miles. Determine the angles of the triangle created by her flight path.

$$\boldsymbol{a = 448, b = 1583, c = 1667}$$

$$\boldsymbol{448^2 = 1583^2 + 1667^2 - 2(1583)(1667)\cos A}$$

$$\boldsymbol{\frac{448^2 - 1583^2 - 1667^2}{-2(1583)(1667)} = \cos A}$$

$$\boldsymbol{0.963 = \cos A}$$

$$\boldsymbol{A \approx 15.6°}$$

$$\boldsymbol{1583^2 = 448^2 + 1667^2 - 2(448)(1667)\cos B}$$

$$\boldsymbol{B = \cos^{-1}\left(\frac{8459}{26672}\right) \approx 71.5°}$$

$$\boldsymbol{C + 15.6 + 71.5 = 180}$$

$$\boldsymbol{C = 92.9°}$$

3. Lola rolls a ball on the ground at an angle of 23° to the right of her dog Buttons. If the ball rolls a total distance of 48 feet, and she is standing 30 feet away from Buttons, how far will Buttons have to run to retrieve the ball?

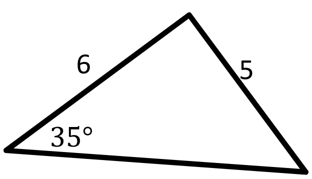
$$\boldsymbol{A = 23°, b = 30, c = 48}$$

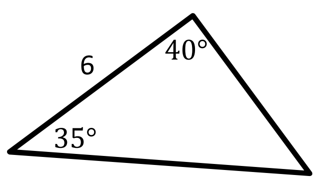
$$\boldsymbol{a^2 = 30^2 + 48^2 - 2(30)(48)\cos 23}$$

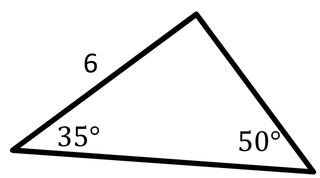
$$\boldsymbol{a = \sqrt{3204 - 2880\cos(23)} \approx 23.5\ feet}$$

4. Let's summarize the Law of Sines and the Law of Cosines.

a. Draw two different triangle situations that you could use the Law of Sines to solve.







b. Draw two different triangle situations that you could use the Law of Cosines to solve.