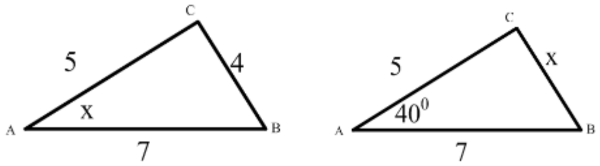
Pre-Calculus Name: Date: Period:

7D.2: The Law of Cosines

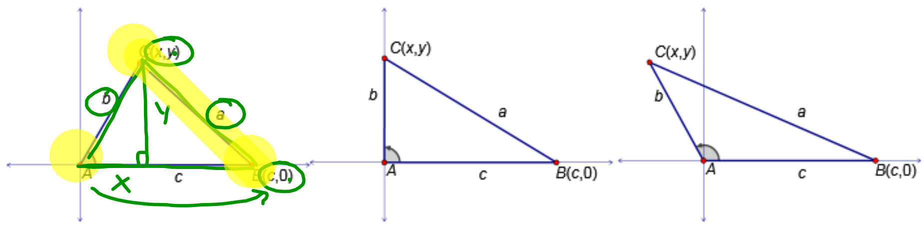
Consider this:

Can we use the Law of Sines to solve for x in these triangles? Explain.

Law of Sines needs 1 Angle-Side Pair



Suppose we make a triangle with a vertex at the origin, choose B on the x-axis, and choose point C above the x-axis. There are three possible types of triangles we can get.



Suppose we know the values of b, c, and $m\angle A$. Can we use the law of sines to solve for a? Since this will not work, we need a new method.

By Definition, we know

$$\cos A = \frac{x}{b}, \qquad \sin A = \frac{y}{b}$$

This gives us

$$x = b\cos A \qquad , \qquad y = b\sin A$$

Using the distance formula to describe the length of a, we get

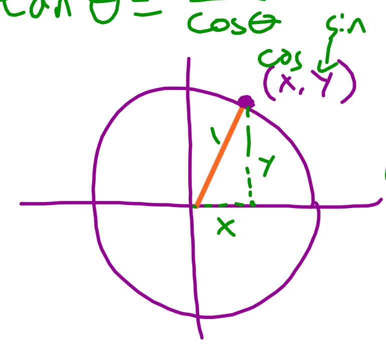
$$a = \sqrt{(x-c)^2 + (y-0)^2}$$

Now, let's substitute and simplify…

$(\cos A)^2 = \cos^2 A$

$$a^2 = (b\cos A - c)^2 + (b\sin A)^2$$
$$a^2 = b^2(\cos A)^2 - 2bc(\cos A) + c^2 + b^2(\sin A)^2$$
$$a^2 = b^2\cos^2 A + b^2\sin^2 A + c^2 - 2bc(\cos A)$$
$$a^2 = b^2\underbrace{(\cos^2 A + \sin^2 A)}_{1} + c^2 - 2bc(\cos A)$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



$x^2 + y^2 = 1^2$

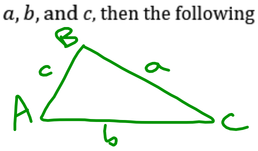
$\boxed{\cos^2\theta + \sin^2\theta = 1}$

The *Pythagorean identity* tells us $\cos^2 A + \sin^2 A = 1$, so we simplify this expression to get

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Sides Angle

Law of Cosines If a triangle has angles $A, B,$ and C with opposite sides of length $a, b,$ and c, then the following must be true.



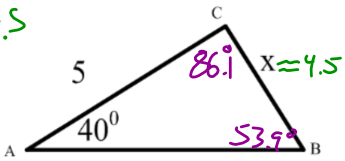
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

When can we use the Law of Cosines? When you know two sides and the included angle (S.A.S.) or when you know three sides (S.S.S.), use the law of cosines to solve the triangle.

Example 1

Use the law of Cosines to find the value of x in the triangles below. Then solve the triangle.

S.A.S



$a^2 = b^2 + c^2 - 2bc(\cos A)$

$x^2 = 5^2 + 7^2 - 2(5)(7)\cos 40$

$x^2 = 20.3738$

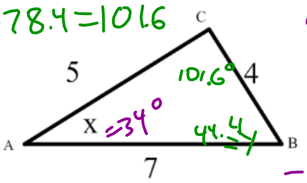
$x \approx 4.51$

$B + 86.1 + 40 = 180$

$\dfrac{\sin 40}{4.51} = \dfrac{\sin C}{7}$

$C = \sin^{-1}\left(\dfrac{7\sin 40}{4.51}\right)$

$180 - 78.4 = 101.6$



$a^2 = b^2 + c^2 - 2bc\cos A$

$4^2 = 5^2 + 7^2 - 2(5)(7)\cos A$

$16 = 74 - 70\cos A$

$-74 \quad -74$

$-58 = -70\cos A$

$\frac{58}{70} = \cos A$

$A = \cos^{-1}\left(\frac{58}{70}\right)$

$A = \cos^{-1}(.8286)$

$A = 34°$

$5^2 = 4^2 + 7^2 - 2(4)(7)\cos B$

$25 = 65 - 56\cos B$

$-40 = -56\cos B$

$\cos^{-1}\left(\frac{40}{56}\right) = B$

$44.4° = B$

Example 2

Use the law of cosines to solve the triangle.

$a = 11, \quad b = 35, \quad c = 15$

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$11^2 = \underbrace{35^2 + 15^2} - \underbrace{2(35)(15)}(\cos A)$

$121 = 1450 - 1050 \cdot \cos A$

$-1450 \quad -1450$

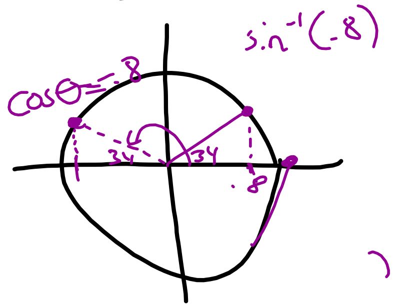
$\dfrac{-1329}{-1050} = \dfrac{-1050 \cdot \cos A}{-1050}$

$1.26 = \cos A$

$\cos^{-1}(1.26) = A$

$\boxed{\text{No Solution}}$

$\sin^{-1}(.8)$



$a^2 = b^2 + c^2 - 2bc(\cos A)$

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Assignment 7D.2: Law of Cosines

1. Solve the following triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

a. $\triangle ABC$, if $A = 42^\circ, b = 12, c = 19$

$a^2 = 12^2 + 19^2 - 2(12)(19)\cos 42$

$a = \sqrt{12^2 + 19^2 - 2(12)(19)\cos 42} \approx 12.89$

$12^2 = 12.89^2 + 19^2 - 2(12.89)(19)\cos B$

b. $\triangle PQR$, if $P = 73^\circ, q = 7, r = 15$

c. $\triangle RST$, if $r = 35, s = 22, and\ t = 25$

d. $\triangle BCD$, if $B = 16^\circ, c = 27,\ d = 3$

2. During her shift, a pilot flies from Columbus to Atlanta, a distance of 448 miles, and then on to Phoenix, a distance of 1583 miles. From Phoenix, she returns home to Columbus, a distance of 1667 miles. Determine the angles of the triangle created by her flight path.

3. Lola rolls a ball on the ground at an angle of 23° to the right of her dog Buttons. If the ball rolls a total distance of 48 feet, and she is standing 30 feet away from Buttons, how far will Buttons have to run to retrieve the ball?

4. Let's summarize the Law of Sines and the Law of Cosines.

a. Draw two different triangle situations that you could use the Law of Sines to solve.

b. Draw two different triangle situations that you could use the Law of Cosines to solve.