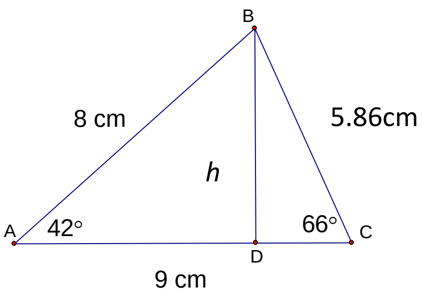
Pre-Calculus Name: Date: Period:

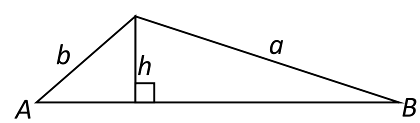
7D.1: The Law of Sines

Discovering the Law of Sines



1. In the triangle to the left, $\overline{BD} \perp \overline{AC}$. Write *two* different equations, one using $\overline{AB}$ and one using $\overline{BC}$, that you could use to find the value of h. Then, solve each equation for h. (note, there may be a small difference due to rounding)

2. Generalize it:
Now, use $\sin A$ and $\sin B$ to write two equations that you could use to solve for h in the triangle to the right.
Solve each of these for h.

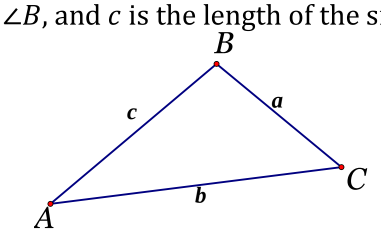


3. Now set the equations in (2) equal to each other. Then change it to a proportion by moving the "A's" to one side and the "B's" to the other. This is called the *Law of Sines*!

Law of Sines:

Suppose that ABC is any triangle(not just a right triangle) with a as the length of the side opposite $\angle A$ b is the length of the side opposite $\angle B$, and c is the length of the side opposite $\angle C$. Then, the following ratios are equal:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

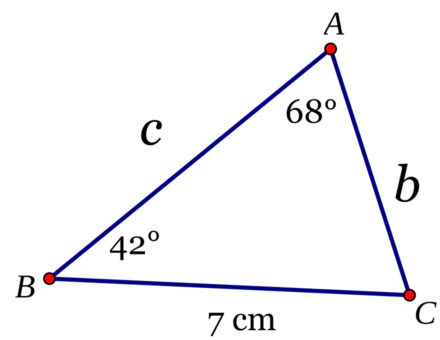


Using the Law of Sines, if we know 3 parts of a triangle, then we can find the other three. There are two cases where this applies:

- **AAS** – We know two angles and a side that is opposite one of these angles.
- **SSA** – We know two sides of a triangle and one angle opposite one of these sides.

Try These:

a) Consider the triangle to the right. (AAS: Angle-Angle-Side)



   i. Use the law of sines equation

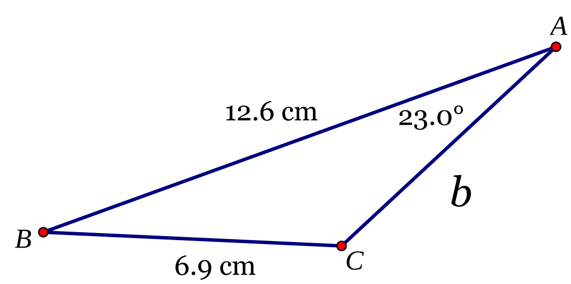
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

   to find the length b to the nearest cm.

   ii. Find $m\angle C$ to the nearest 10th using the other two angle measures.

   iii. Use the law of sines again to solve for the length c to the nearest cm.

b) Consider the triangle to the right with two known sides and one angle. (SSA: Side-Side-Angle)



   i. Find $m\angle C$ to the nearest 10th using the law of sines equation $\frac{\sin A}{a} = \frac{\sin C}{c}$

Watch out! Does your answer match the drawing? You should have found an acute angle, but the drawing is an obtuse angle for $\angle C$. Is the another angle that has a $\sin C = .7326$?

This is called the **ambiguous case** in which there are two possible answers for $m\angle C$ (this happens when we have *Side-Side-Angle* given to us):

$$m\angle C = 47.1°, or\ m\angle C = 180 - 47.1 = 132.9°$$

   ii. Find $m\angle B$ using the other angle measures.

   iii. Find the length b using $m\angle B$ you just found.