Pre-Calculus Name: SOLUTIONS Date: Period:

Assignment 7C: The Unit Circle

Answer the following problems from your Lippman/Rasmussen textbook with as much detail, explanation, and work that is appropriate.

5.3: 3, 7, 12, 14, 15

3. The point P is on the unit circle. If the y-coordinate of P is $\frac{3}{5}$, and P is in quadrant II, find the x coordinate.

Because sine is the x-coordinate divided by the radius, we have $\sin\theta = \left(\frac{3}{5}\right)/1$ or just $\frac{3}{5}$. If we use the trig version of the Pythagorean theorem, $\sin^2\theta + \cos^2\theta = 1$, with $\sin\theta = \frac{3}{5}$, we get $\frac{9}{25} + \cos^2\theta = 1$, so $\cos^2\theta = \frac{25}{25} - \frac{9}{25}$ or $\cos^2\theta = \frac{16}{25}$; then $\cos\theta = \pm\frac{4}{5}$. Since we are in quadrant 2, we know that $\cos\theta$ is negative, so the result is $-\frac{4}{5}$.

7. If $\sin(\theta) = \frac{3}{8}$ and θ is in the 2nd quadrant, find $\cos(\theta)$.

If $\sin\theta = \frac{3}{8}$ and $\sin^2\theta + \cos^2\theta = 1$, then $\frac{9}{64} + \cos^2\theta = 1$, so $\cos^2\theta = \frac{55}{64}$ and $\cos\theta = \frac{\pm\sqrt{55}}{8}$; in the second quadrant we know that the cosine is negative so the answer is $-\frac{\sqrt{55}}{8}$

12. For each of the following angles, find the reference angle and which quadrant the angle lies in. Then compute sine and cosine of the angle.

a. $\frac{4\pi}{3}$ $\frac{\pi}{3}$, *III quadrant,* $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$, $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

b. $\frac{2\pi}{3}$ $\frac{\pi}{3}$, *II quadrant,* $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$, $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

c. $\frac{5\pi}{6}$ $\frac{\pi}{6}$, *II quadrant,* $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

d. $\frac{7\pi}{4}$ $\frac{\pi}{4}$, **IV quadrant,** $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

14. Give exact values for $\sin(\theta)$ and $\cos(\theta)$ for each of these angles.

a. $-\frac{2\pi}{3}$ b. $\frac{17\pi}{4}$ c. $-\frac{\pi}{6}$ d. 10π

$$\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2},$$
$$\sin\left(\frac{17\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{17\pi}{4}\right) = \frac{\sqrt{2}}{2},$$
$$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}, \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$
$$\sin(10\pi)=0,\ \cos(10\pi) = 1$$

15. Find an angle θ with $0 < \theta < 360°$ or $0 < \theta < 2\pi$ that has the same sine value as:

a. $\frac{\pi}{3}$ b. 80°   c. 140°   d. $\frac{4\pi}{3}$ e. 305°

a. $\frac{\pi}{3}$ is in quadrant 1, where sine is positive; if we choose an angle with the same reference angle as $\frac{\pi}{3}$ but in quadrant 2, where sine is also positive, then it will have the same sine value. $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, so $\frac{2\pi}{3}$ **has the same reference angle and sine as $\frac{\pi}{3}$.**

b. Similarly to problem a. above, 100° = 180° - 80° , so both 80° and 100° have the same reference angle (80°) , and both are in quadrants where the sine is positive, so 100° has the same sine as 80°.

c. 140° is 40° less than 180°, so its reference angle is 40°. It is in quadrant 2, where the sine is positive; the sine is also positive in quadrant 1, so **40° has the same sine value and sign as 140°.**

d. $\frac{4\pi}{3}$ is $\frac{\pi}{3}$ more than π, so its reference angle is $\frac{\pi}{3}$. It is in quadrant 3, where the sine is negative. Looking for an angle with the same reference angle of $\frac{\pi}{3}$ in a different quadrant where the sine is also negative, **we can choose quadrant 4 and $\frac{5\pi}{3}$** which is $2\pi - \frac{\pi}{3}$.

e. 305° is 55° less than 360°, so its reference angle is 55° . It is in quadrant 4, where the sine is negative. An angle with the same reference angle of 55° in quadrant 3 where the sine is also negative **would be 180° + 55° = 235°.**

Compute the following exactly.

a. $\csc\left(\frac{\pi}{6}\right) = 2$ b. $\tan\left(\frac{\pi}{4}\right) = 1$ c. $\sec\left(\frac{7\pi}{4}\right) = \sqrt{2}$ d. $\cot\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{3}$