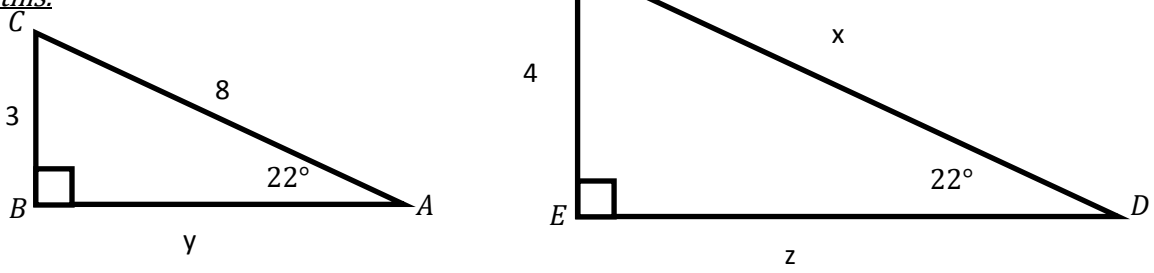
7B: Six Trigonometric Functions

In the study of geometry, the concept of similar triangles plays an important role in connecting geometric concepts to algebraic functions.

Def: ***Similar triangles*** have congruent angles and proportional sides.

We can prove that if *two angles* of one triangle are congruent to *two angles* of another triangle, then the triangles are similar (the "Angle-Angle Similarity" postulate). This works nice for right triangles.

Consider this.



A. Are the triangles above similar? Explain.

B. Use a proportion to find the value of x.

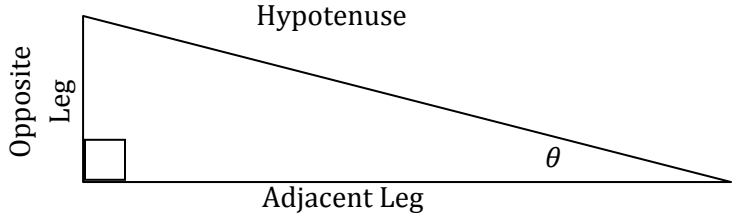
C. Use the names of the sides to complete the proportion:

$$\frac{BC}{AC} = \frac{}{DF}$$

D. Write 5 more true proportions like the one in (c). Write each ratio using sides from the same triangle.

The Trig. Ratios

By the Angle-Angle Similarity theorem for triangles, if two right triangles have one acute angle congruent, then they are similar. Since they are similar, there are six ratios that must be equal for the two triangles. These are called the **trigonometric ratios** or **trigonometric functions.**



Trigonometric Functions

$$Sine(\theta) = \sin\theta = \frac{opp}{hyp} \qquad Cosecant(\theta) = \csc\theta = \frac{hyp}{opp}$$

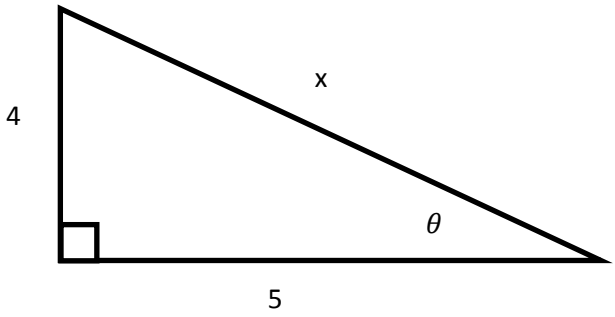
$$Cosine(\theta) = \cos\theta = \frac{adj}{hyp} \qquad Secant(\theta) = \sec\theta = \frac{hyp}{adj}$$

$$Tangent(\theta) = \tan\theta = \frac{opp}{adj} \qquad Cotangent(\theta) = \cot\theta = \frac{adj}{opp}$$

Example

a) Find the exact value of x.

b) Find the value of the six trigonometric functions for the angle θ



Example

a) If $\tan\theta = \frac{5}{7}$, find the value of the other trig functions.

Explore

a) Which ratios are reciprocally of each other?
b) Which trig function is equal to $\frac{\sin\theta}{\cos\theta}$?
c) Which trig ratio is equal to $\frac{\csc\theta}{\cot\theta}$?
d) Which trig function must be less than 1?
e) Which trig functions must be greater than 1?

Trigonometric Ratios and Your Calculator

Since trigonometric ratios of a given angle are constant, your calculator has been programmed to remember these values in decimal form (some newer calculators will actually give the exact radical version of certain angles.) However, you must ***make sure you are in the correct mode!*** All scientific and graphing calculators can be put into degree mode or radian mode, and some can measure in "gradians" ($400\ grad = 360° = 2\pi\ rad$).

Your calculator can evaluate sine, cosine, and tangent. To find the reciprocal functions, you need to use the identities:

$$\csc\theta = \frac{1}{\sin\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \cot\theta = \frac{1}{\tan\theta}$$

For Example: $\csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} \approx 1.4142$

Try it!

Use your calculator to evaluate the following to four decimal places.

a) $\sin 30°$

b) $\csc 30°$

c) $\cos\left(\frac{\pi}{5}\right)$

d) $\sec\left(\frac{2\pi}{3}\right)$

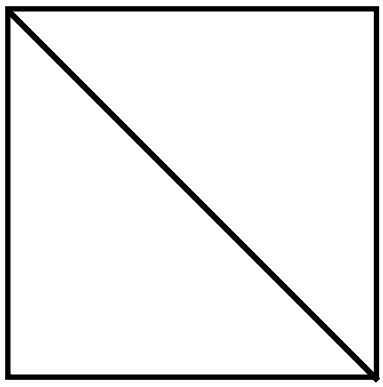
e) $\tan(54°32'12")$
(use the "angle" menu for minutes and seconds)

f) $\cot(35°30')$

Special right triangles

Some angles are vital to work in trigonometry and calculus. These angles come from half of a square and half of a Equilateral Triangle

Explore For the equilateral triangle and square below, find the length of each segment in terms of a and the measure of all the angles in radians. Label these on the figures.



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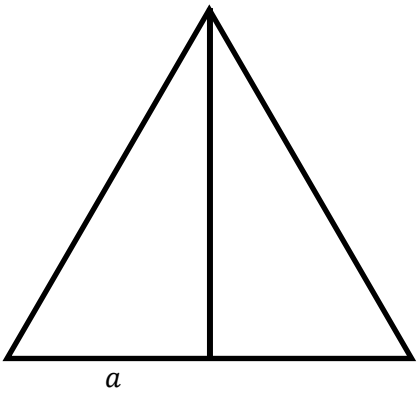

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Use your results from above to find the following

$\sin\left(\frac{\pi}{4}\right) =$	$\csc\left(\frac{\pi}{4}\right) =$	$\sin\left(\frac{\pi}{6}\right) =$	$\csc\left(\frac{\pi}{6}\right) =$	$\sin\left(\frac{\pi}{3}\right) =$	$\csc\left(\frac{\pi}{3}\right) =$
$\cos\left(\frac{\pi}{4}\right) =$	$\sec\left(\frac{\pi}{4}\right) =$	$\cos\left(\frac{\pi}{6}\right) =$	$\sec\left(\frac{\pi}{6}\right) =$	$\cos\left(\frac{\pi}{3}\right) =$	$\sec\left(\frac{\pi}{3}\right) =$
$\tan\left(\frac{\pi}{4}\right) =$	$\cot\left(\frac{\pi}{4}\right) =$	$\tan\left(\frac{\pi}{6}\right) =$	$\cot\left(\frac{\pi}{6}\right) =$	$\tan\left(\frac{\pi}{3}\right) =$	$\cot\left(\frac{\pi}{3}\right) =$