Eccentricity: Ellipses have many applications in science. One of the most important results involving ellipses is Kepler's first law of planetary motion that states that a planet's orbit is an ellipse with the Sun as one of the foci. The term used term used to describe the shape of an elliptical orbit is the *eccentricity* defined as

$$\textit{Eccentricity: } e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

where a is the semimajor axis, b is the semiminor axis, and c is the focal distance.

Example Find the *eccentricity* of the ellipse in the previous example.

$$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$$

Example The earth has a semimajor axis $a \approx 149.59\ Gm$ (gigameters=$10^9 m$), and a semiminor axis $b \approx 149.577$. Find the eccentricity.

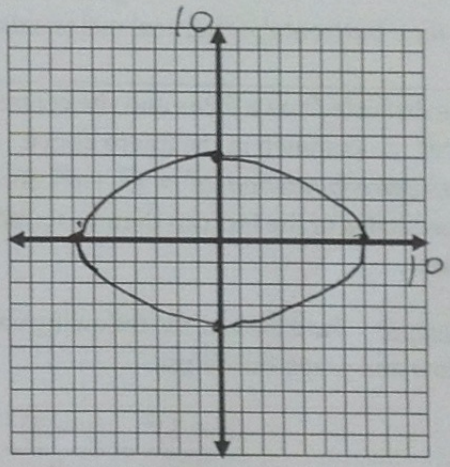
Practice Problems

For each of the ellipses below:

a) Find the length of the semimajor (a) and semiminor (b) axes.
b) Find the focal distance (c) and the eccentricity (e) of the ellipse.
c) Find the coordinates of the vertices and foci.
d) Graph the ellipse.

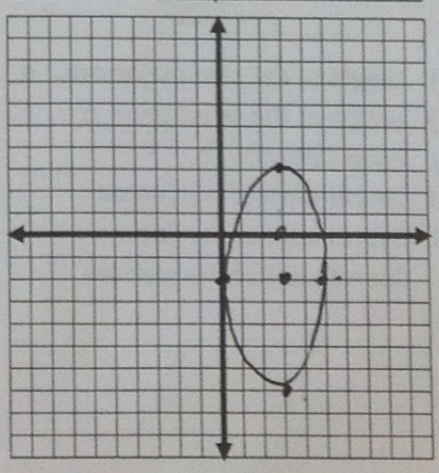
1. $\frac{x^2}{49} + \frac{y^2}{16} = 1$

$a=7$ $C(0,0)$
$b=4$ $V_1(0,4)$ $V_2(0,-4)$
$c=\sqrt{33}$ $V_3(7,0)$ $V_4(-7,0)$
$e=\frac{\sqrt{33}}{7}$ $F_1(\sqrt{33},0)$ $F_2(-\sqrt{33},0)$

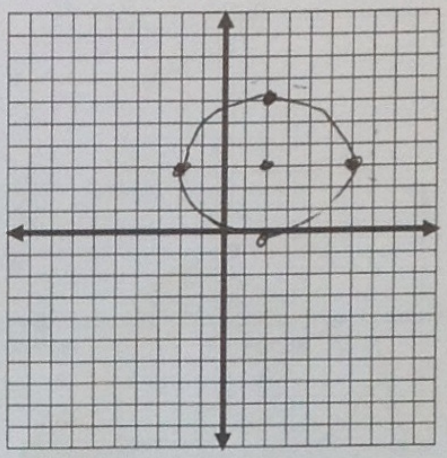


2. $\frac{(x-3)^2}{4} + \frac{(y+2)^2}{25} = 1$

$a=5$ $C(3,-2)$
$b=2$ $V_1(5,-2)$ $V_2(1,-2)$
$c=\sqrt{21}$ $V_3(3,-7)$ $V_4(3,3)$
$e=\frac{\sqrt{21}}{5}$ $F_1(3,-2+\sqrt{21})$
$F_2(3,-2-\sqrt{21})$

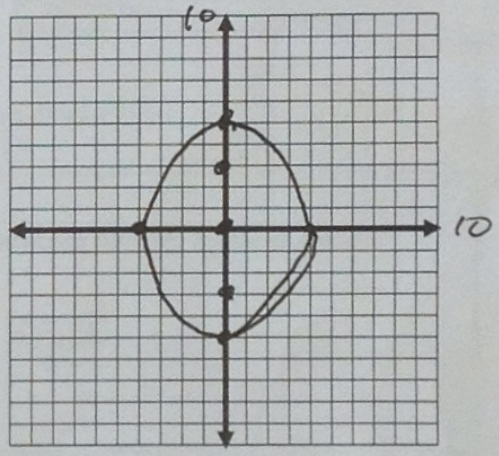


3. Consider an ellipse with a center at (2,3), and vertices at (2,6) and (6,3). Sketch the ellipse and find its equation.



$$\frac{(x-2)^2}{16} + \frac{(y-3)^2}{9} = 1$$

4. Consider an ellipse with a foci at (0,3) and (0, −3) and a semimajor axis length $a = 5$. Sketch the ellipse and find its equation.



$c(0,0)$ $c=3$ $a=5$

$a^2 = b^2 + c^2$

$5^2 = b^2 + 3^2$ $b=4$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

5. Mercury's orbit has a semimajor axis of 57.9 Gm and an Eccentricty of $e \approx .2056$. Find the focal distance c, and the length of the semiminor axis b.

$a^2 = b^2 + c^2$

$57.9^2 = b^2 + 11.9^2$

$e = \frac{c}{a}$

$.2056 = \frac{c}{57.9}$ $c = 11.9$ $56.66 = b$

6. Use the information from #6 to write an equation for Mercury's orbit in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{3210.8} + \frac{y^2}{3352.4} = 1$$

7. Pluto has an eccentricity of $e \approx 0.2484$. Which planet has an orbit that is closer to a circle, Mercury or Pluto?

Mercury, because its eccentricity is closer to 0.