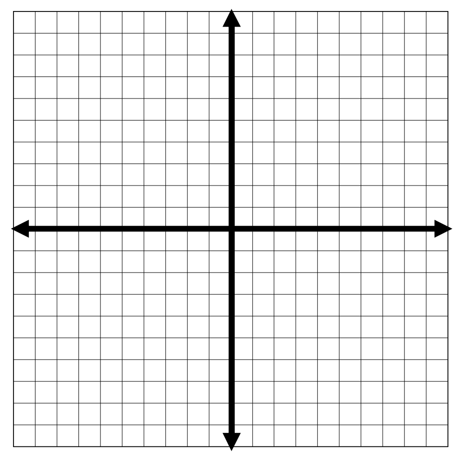
6.1B-Hyperbolas

Exploring Hyperbolas

We have now worked with the graphs of circles ($x^2 + y^2 = r^2$) and the graphs of ellipses $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$ which are closely related since an ellipse degenerates into a circle. Let's now consider a similar equation to these by changing the addition sign in the equation.

1. Consider the equation $x^2 - y^2 = 1$.
   a. Complete the table by substituting for x and solving for y. (Don't forget positive and negative square roots.)

x	y
3	
2	
1	
0	
-1	
-2	
-3	



   b. What happened when $x = 0$? Are there any solutions when $-1 < x < 1$?

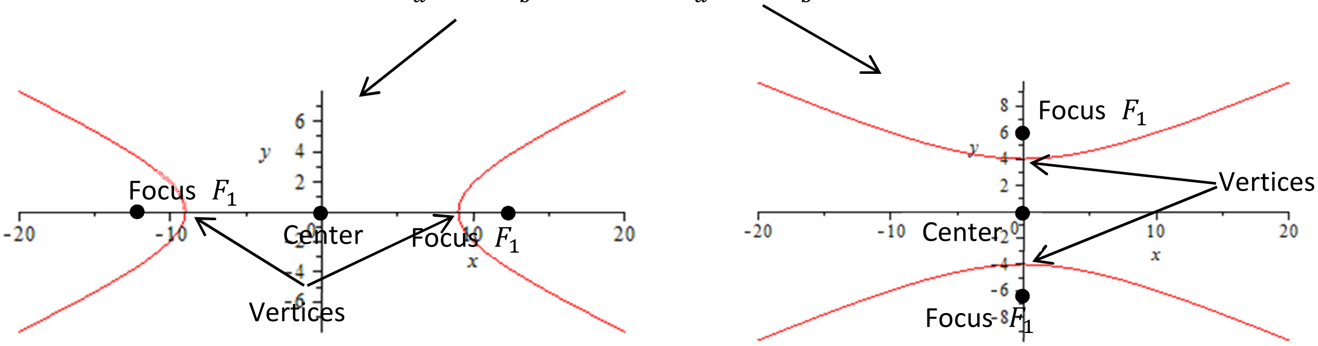
   c. Plot the points and draw the graph

   d. As $x \to \infty$, what lines does the graph approach?

Parts of a Hyperbola

The standard form of a hyperbola is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ or } \frac{(y-h)^2}{a^2} - \frac{(x-k)^2}{b^2} = 1$$



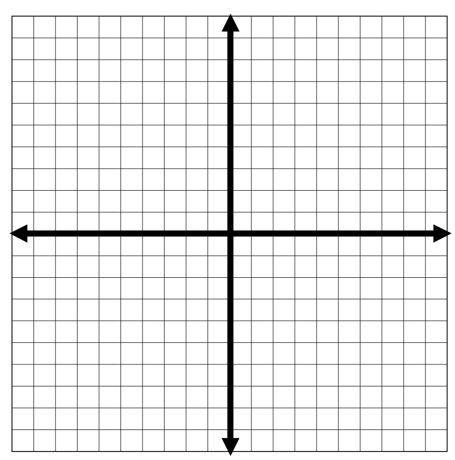
The line through the foci is called the ***focal axis***, the chord connecting the foci is called the ***transverse axis***, and the perpendicular bisector of the transverse axis is called the ***conjugate axis***.

We will use the **Pythagorean relation** $c^2 = a^2 + b^2$

	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-h)^2}{a^2} - \frac{(x-k)^2}{b^2} = 1$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Asymptotes	$y = \pm \frac{b}{a} x$	$y = \pm \frac{a}{b} x$

<u>Example</u>

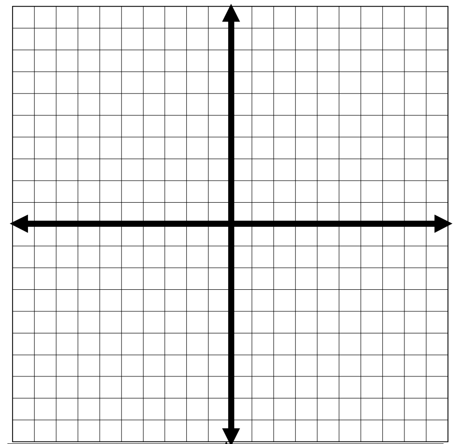
Graph $\frac{x^2}{4} - \frac{y^2}{9} = 1$



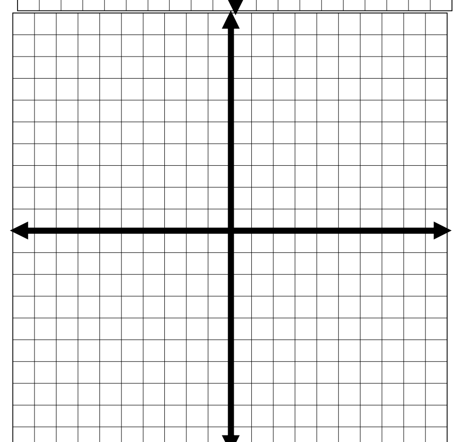
Practice Problems

For each of the ellipses below,

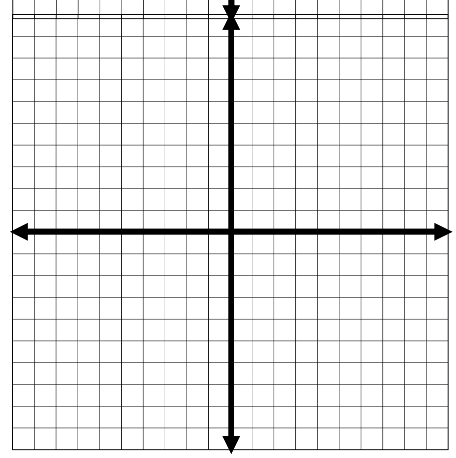
a) Find the coordinates of the vertices and the foci
b) Draw the asymptotes
c) And graph the hyperbola.



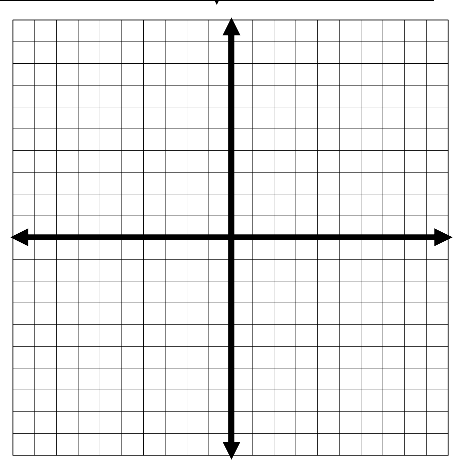
1. $\frac{x^2}{9} - \frac{y^2}{16} = 1$



2. $\frac{y^2}{49} - \frac{x^2}{4} = 1$



3. $3x^2 - 4y^2 = 12$
   (Hint: Divide First)



4. $\frac{(x+1)^2}{9} - \frac{(y-3)^2}{16} = 1$