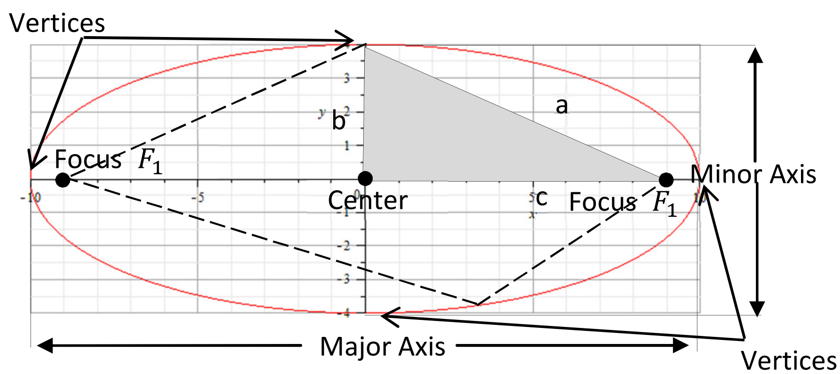
Pre-Calculus Name: Date: Period:

6.1A-More on Ellipses

Ellipses

An ellipse is the set of all points in a plane whose distances from *two fixed points (called the "**foci**" – plural for focus)* have a constant sum.



The **major axis** is the longest chord of the ellipse, and the **minor axis** is chord on the perpendicular bisector of the major axis. Half of these axes are called the ***semimajor axis*** and the ***semiminor axis.*** These values come from the square root of the denominators of the standard form equation of an ellipse.

Standard Form Equation of an Ellipse with center (h, k):

If an ellipse has a center at (h, k), a semimajor axis of a, and a semiminor axis of b, the standard equation is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \qquad or \quad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,$$

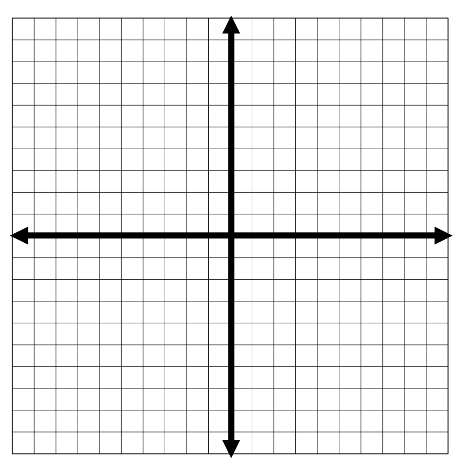
for a horizontal or vertical major axis respectively.

The distance from a focus and the center of the ellipse is called the **focal distance**, typically denoted with the letter c as shown in the figure above. The focal distance of the ellipse can be found using the *Pythagorean relation* $a^2 = b^2 + c^2$ for a semimajor length a and semiminor length b. The coordinates of the foci can then be found by adding the focal distance to the appropriate coordinate of the center.

Example Find each of the following values for the ellipse below and graph the ellipse.

$$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$$

a) Length of the semimajor axis, a.
b) Length of the semiminor axis, b.
c) Focal Distance, c.
d) Coordinates of the Center
e) Coordinates of the Foci.
f) Coordinates of the vertices



Eccentricity: Ellipses have many applications in science. One of the most important results involving ellipses is Kepler's first law of planetary motion that states that a planet's orbit is an ellipse with the Sun as one of the foci. The term used term used to describe the shape of an elliptical orbit is the ***eccentricity*** defined as

$$Eccentricity\colon\ e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

where a is the semimajor axis, b is the semiminor axis, and c is the focal distance.

Example Find the *eccentricity* of the ellipse in the previous example.

$$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$$

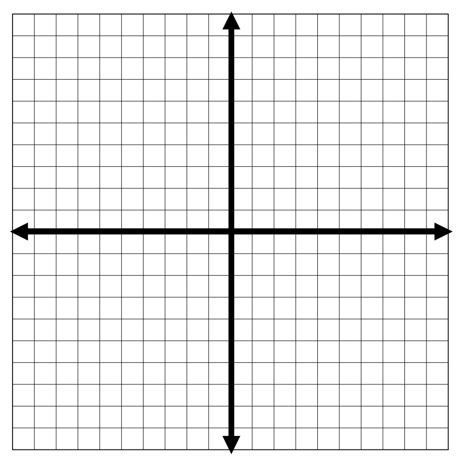
Example The earth has a semimajor axis $a \approx 149.59\ Gm$ (gigameters=$10^9 m$), and a semiminor axis $b \approx 149.577$. Find the eccentricity.

Practice Problems

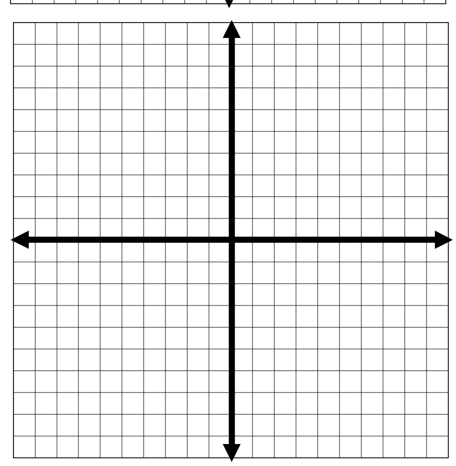
For each of the ellipses below:

a) Find the length of the semimajor (a) and seminor (b) axes.
b) Find the focal distance (c) and the eccentricity (e) of the ellipse.
c) Find the coordinates of the vertices and foci.
d) Graph the ellipse.

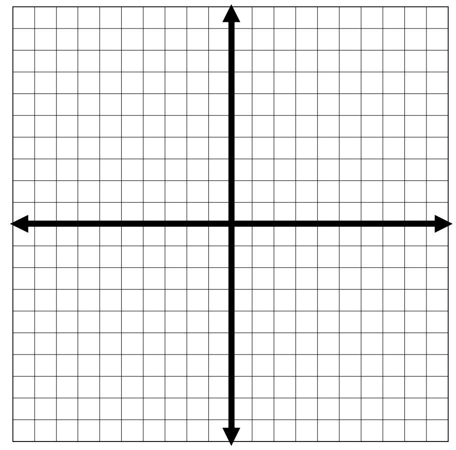
1. $\frac{x^2}{49} + \frac{y^2}{16} = 1$



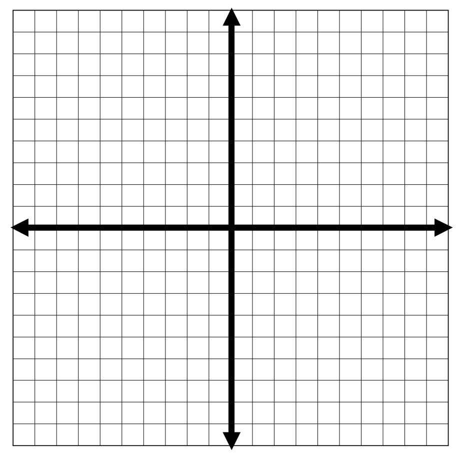
2. $\frac{(x-3)^2}{4} + \frac{(y+2)^2}{25} = 1$



3. Consider an ellipse with a center at $(2,3)$, and vertices at $(2,6)$ and $(6,3)$. Sketch the ellipse and find its equation.



4. Consider an ellipse with a foci at $(0,3)$ and $(0,-3)$ and a semimajor axis length $a = 5$. Sketch the ellipse and find its equation.



5. Mercury's orbit has a semimajor axis of $57.9\ Gm$ and an Eccentricty of $e \approx .2056$. Find the focal distance c, and the length of the semiminor axis b.

6. Use the information from #6 to write an equation for Mercury's orbit in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

7. Pluto has an eccentricity of $e \approx 0.2484$. Which planet has an orbit that is closer to a circle, Mercury or Pluto?