Unit 5Practice Test

## SOLUTIONS

## Learning Targets: 5A-5C

## Complete the problems below, show your work, and write your answer in the blank provided.

Target 5A
I can solve and apply systems of nonlinear equations in two variables.

1. Solve system of equations by substitution. (Show your work!)

$$
\begin{gathered}
y=x^{2} \\
y-9=0 \\
(x=-3, y=9) \text { or }(x=3, y=9)
\end{gathered}
$$

2. Solve the system of equations by elimination. (Show your work!)

$$
\begin{gathered}
3 x+5 y=7 \\
4 x-2 y=-3 \\
\left(x=-\frac{\mathbf{1}}{\mathbf{2 6}}, y=\frac{\mathbf{3 7}}{\mathbf{2 6}}\right)
\end{gathered}
$$

3. a. Solve the following system of equations algebraically.

$$
\begin{gathered}
x^{2}+y^{2}=4 \\
x+y=2 \\
(x=2, y=0) \text { or }(x=0, y=2)
\end{gathered}
$$

b. Check your answer graphically. Label your solution(s) on the graph.


## Target 5B

I can solve and apply systems of linear equations in three variables using substitution, elimination, and matrices.
4. Solve the following system of equations using the given method. (You must show work to get credit.)
a) Use substitution.

$$
\begin{aligned}
& 2 x+y+z=3 \\
& x-y+2 z=0 \\
& x=y+4
\end{aligned}
$$

$$
\begin{aligned}
& \left\{\begin{array} { l } 
{ 2 ( y + 4 ) + y + z = 3 } \\
{ ( y + 4 ) - y + 2 z = 0 }
\end{array} \rightarrow \left\{\begin{array} { c } 
{ 3 y + z = - 5 } \\
{ 2 z = - 4 }
\end{array} \rightarrow \left\{\begin{array} { c } 
{ 3 y - 2 = - 5 } \\
{ z = - 2 }
\end{array} \rightarrow \left\{\begin{array}{l}
y=-1 \\
z=-2
\end{array}\right.\right.\right.\right. \\
& x=(-1)+4=3 \\
& \text { Solution: }(-1,-2,3)
\end{aligned}
$$

b) Use elimination.
$x+2 y-z=3$
$3 x+7 y-3 z=12$
$-2 x-4 y+3 z=-5$
$\left\{\begin{array}{c}x+2 y-z=3 \\ 3 x+7 y-3 z=12 \\ -2 x-4 y+3 z=-5\end{array} \rightarrow\left\{\begin{array}{c}6 x+12 y-6 z=18 \\ 6 x+14 y-6 z=24 \\ -6 x-12 y+9 z=-15\end{array} \rightarrow\left\{\begin{array}{r}6 x+12 y-6 z=18 \\ -2 y=-6 \\ 3 z=3\end{array} \rightarrow\left\{\begin{array}{r}x+2(3)-(1)=3 \\ y \quad=3 \\ z=1\end{array}\right.\right.\right.\right.$

## Solution: $(-2,3,1)$

5. Solve the following system of equations using an augmented matrix and your calculator. Show all the steps needed to solve this by hand using matrices.

$$
\begin{aligned}
& x-2 y=0 \\
& -2 x+3 y=-2 \\
& \qquad\left[\begin{array}{cc|c}
1 & -2 & 0 \\
-2 & 3 & -2
\end{array}\right]=\left[\begin{array}{cc|c}
2 & -4 & 0 \\
-2 & 3 & -2
\end{array}\right]=\left[\begin{array}{cc|c}
2 & -4 & 0 \\
0 & -1 & -2
\end{array}\right]=\left[\left.\begin{array}{cc}
2 & -4 \\
0 & 4
\end{array} \right\rvert\, \begin{array}{l}
0 \\
8
\end{array}\right]=\left[\begin{array}{cc|c}
2 & 0 & 8 \\
0 & 4 & 8
\end{array}\right]=\left[\begin{array}{ll}
1 & 0 \\
0 & 1 \\
2
\end{array}\right]
\end{aligned}
$$

$$
\text { Solution: }(4,2)
$$

6. Solve the following system of equations using a matrix equation and your calculator. (You must write the matrix equation to receive full credit.)
$x-2 y+z=-2$
$2 x-3 y+2 z=2$
$4 x-8 y+5 z=-5$

$$
\begin{gathered}
{\left[\begin{array}{ccc|c}
1 & -2 & 1 & -2 \\
2 & -3 & 2 & 2 \\
4 & -8 & 5 & -5
\end{array}\right]=\left[\begin{array}{ccc|c}
1 & 0 & 0 & 7 \\
0 & 1 & 0 & 6 \\
0 & 0 & 1 & 3
\end{array}\right]} \\
\text { Solution: }(7,6,3)
\end{gathered}
$$

7. Solve the following system of equations using the method of your choice.

$$
\begin{aligned}
& x-2 y+z-w=2 \\
& 2 x+y-z=-1 \\
& x-y+2 z-w=-1 \\
& x+3 y-z+w=4 \\
& \begin{array}{c}
{\left[\begin{array}{cccc|c}
\mathbf{1} & -2 & 1 & -1 & 2 \\
2 & 1 & -1 & 0 & -1 \\
1 & -1 & 2 & -1 & -1 \\
1 & 3 & -1 & 1 & 4
\end{array}\right]=\left[\begin{array}{cccc|c}
\mathbf{1} & 0 & 0 & 0 & 8 \\
0 & 1 & 0 & 0 & -10 \\
0 & 0 & 1 & 0 & 7 \\
0 & 0 & 0 & 1 & 33
\end{array}\right]} \\
\text { Solution: }(\boldsymbol{x}=\mathbf{8}, \boldsymbol{y}=-\mathbf{1 0}, \mathbf{z}=\mathbf{7}, \boldsymbol{w}=\mathbf{3 3})
\end{array}
\end{aligned}
$$

## Target 5C

I can solve and apply systems of nonlinear inequalities in two variables.
8. Solve the following system of inequalities.
(Label your axes!)
$5 x-3 y>1$
$3 x+4 y \leq 18$

9. Solve the following system of inequalities. (Label your scales!)

$$
\begin{aligned}
& y \geq x^{2} \\
& x^{2}+y^{2} \leq 4
\end{aligned}
$$

10. Write the system of inequalities for the system shown below.

$$
\begin{aligned}
& x^{2}+y^{2} \leq 25 \\
& y \geq 2 x+1
\end{aligned}
$$


11. Write a system of inequalities that would be needed to represent a metal washer (shaped like a donut) that has an outside radius of 8 cm and an inside radius of 3 cm .


$$
\begin{aligned}
& x^{2}+y^{2} \leq 64 \\
& x^{2}+y^{2} \geq 9
\end{aligned}
$$

12. Checking Eric!

Eric thinks that $(3,1)$ is a solution to the system of inequalities shown below. Explain why Eric is right or wrong. Use a graph to help with your justification.
$y>x-2$
$2 y<-4 x+4$

Eric is wrong because the point $(3,1)$ isn't in the double shaded region

13. Hank can row a boat 2 miles upstream (against the current) in 45 minutes. He can row the same distance downstream in 30 minutes. If both the rowing speed and current speed are constant, find Hank's rowing speed and the speed of the current.

$$
\begin{gathered}
2=(x-y) \cdot 75 \\
2=(x+y) \cdot .5 \\
(x=3.33, y=.67) \\
\text { Rowing Speed }=3.33 \mathrm{mph}, \quad \text { current speed }=.67 \mathrm{mph}
\end{gathered}
$$

14. Diane is going to make a 10 liter mixture that is $35 \%$ salt. To do this, she is going to combine to different saline solutions, one that is $55 \%$ salt and the other that is $20 \%$ salt. How many liters of each type of solution should she mix together to get the 10 liter solution that is $35 \%$ salt? Round your answer to the nearest tenth of a liter. If you use matrices, then show your set up.

$$
\begin{gathered}
x+y=10 \\
.55 x+.20 y=.35(10) \\
\left(x=\frac{30}{7}, y=\frac{40}{7}\right) \\
\text { Liters of } 55 \%=\frac{30}{7} l \\
\text { Liters of } 20 \%=\frac{40}{7} l
\end{gathered}
$$

