5C-2: Applications of Systems of Equations

Let's now look at a couple applications of systems of equations.

Combined Rate Problems

When working with problems dealing with distance (d), time (t), and rate (r) (or speed), we must remember the important formula:

$$d = r \cdot t.$$

However, we sometimes have more than one force adding(or subtracting) to the overall rate as the speed of the river and the motor in the example below. This gives us a system of equations.

Example

Tom Sawyer drove a river boat at a constant speed up the Mississippi against the current for 2 hours and went 10 miles. He then turned the riverboat around and drove the riverboat at the same constant speed for10 miles with the current, but it only took .5 hours.

Find the speed of the boat in still water and find the speed of the river.

Solution:

Step 1: Define Variables

Let $x = speed\ of\ boat\ (in\ still\ water)$

Let $y = speed\ of\ current$

Step 2: Write two equations from given information

From the first trip (using $d = r \cdot t$):

$$10 = (x - y) \cdot 2$$

From the second trip (using $d = r \cdot t$):

$$10 = (x + y) \cdot .5$$

Step 3: Solve system

$$E_1\colon\ 10 = (x - y) \cdot 2$$
$$E_2\colon 10 = (x + y) \cdot .5$$

Let's use elimination. Simplifying these equations and multiplying the second equation by 4 gives us

$$E_1\colon\ 10 = 2x - 2y$$
$$E_2\colon\ 40 = 2x + 2y$$

Adding these equations gives us

$$E_3\colon\ 50 = 4x.$$

So, $x = 12.5\ mph$. Now, we can substitute to get

$$10 = 2(12.5) - 2y$$
$$10 = 25 - 2y$$
$$-15 = -2y$$
$$y = 7.5\ mph$$

So, the speed of the boat is 12.5 mph in still water and the speed of the current is 7.5 mph.

Mixture Problems

Chemistry Background:

A mixture is a combination that contains two or more identifiable substances that do not share any chemical bonds. If one substance is dissolved in another, we call it a solution. For example, if you pour water and sand (or salt and sand) into a container you would have a mixture because the two substances would mix together, but they could be separated by non-chemical processes.

A solution is often identified by its concentration. For example a solution with a concentration of 15% Sodium Chloride ($NaCl$) is 15% Sodium Chloride (which is called the *"solute"*) and 85% water (H_2O, which is called the *"solvent"*).

If we know the volume and concentration, we can find the volume of the solute using the formula:

$$Solution\ concentration \cdot Solution\ volume = Solute\ volume.$$

For example if we have 12 ml of 25% Sodium Chloride solution, the volume of Sodium Chloride (*a.k.a. table salt*) is

$$Volume\ of\ NaCl = .25 \cdot 12 = 3\ ml.$$

Mathematics Problems:

Example

Suppose we have two large bottles of hydrochloric acid solution (a mixture of water, H_2O, and hydrochloric acid, HCl). Bottle #1 is 15% HCl (which means that 85% of the mixture is water), and bottle #2 is 25% HCl. How much of each solution do we need to acquire a 50 ml of 18% HCl?

Step 1: Define Variables

Let $x = volume\ of\ the\ 15\%\ HCl$

Let $y = volume\ of\ 25\%\ HCl$

Step 2: Write a total volume equation

We know that

$$Volume\ of\ the\ 15\%\ HCl + Volume\ of\ 25\%\ HCl = Total\ Volume$$

Since we want a total of 50 ml, we can write the equation as

$$x + y = 50$$

Step 3: Write the concentration equation.

We know the volume of HCl mixed together gives us the total volume of HCl, so we have the equation

$$Vol.of\ HCl\ from\ Bottle\ \#1 + Vol.of\ HCl\ from\ Bottle\ \#2 = Total\ Vol.of\ HCl.$$

From the defined variables and problem information we know

$$Vol.of\ HCl\ from\ Bottle\ \#1 = .15x$$
$$Vol.of\ HCl\ from\ Bottle\ \#2 = .25y.$$

Since we want 50 ml of 18% solution, we know

$$Total\ Vol.of\ HCl = .18(50).$$

Rewriting the generic concentration equation above, we

$$.15x + .25y = .18(50).$$

Step 4: Write and solve the system

We now have the following system of equations:

$$\begin{aligned} x + y &= 50 \\ .15x + .25y &= .18(50) \end{aligned}$$

We can solve this using any substitution, elimination, or matrices. Let's try elimination:

Dividing the second equation by -.15 gives us

$$\begin{aligned} x + y &= 50 \\ -x - \frac{5}{3}y &= -60 \end{aligned}$$

Adding the equation gives us

$$\begin{aligned} -\frac{2}{3}y &= -10 \\ y &= 15. \end{aligned}$$

Substituting this value into the first equation gives us

$$\begin{aligned} x + (15) &= 50 \\ x &= 35. \end{aligned}$$

Solution: So, we conclude that we need 35 ml of the 15% solution and 15 ml of the 25% solution.

5C-2 Assignment

1. Caleb rows a boat upstream in a river for 3 miles in 90 minutes. He then turns around and rows the same distance downstream in only 1 hour. If both the rowing speed and current speed are constant, find Caleb's rowing speed and the speed of the current.

2. An airplane travels 45 minutes into a headwind (against the wind) for 100 miles. After dropping off some passengers, it travels the same distance with the same wind that is now a tailwind (the wind pushes the plane from behind) in only 20 minutes. If both the wind speeds and the airplane speeds are constant, find the speed of the airplane (in still air) and the speed of the wind.

3. Suppose we have two large bottles of hydrochloric acid solution. Bottle A is 5% HCl and bottle B is 35% HCl. How much of each solution do we need to acquire a 50 ml of 15% HCl?

4. Suppose we have two large bottles of saline solution (a mixture of water, H_2O, and sodium chloride, $NaCl$). Bottle A is 10% $NaCl$, and bottle B is 20% $NaCl$. How much of each solution do we need to acquire a 10 ml of 13% $NaCl$?

5. The molar mass of a compound is the mass of one mole of that compound measured in atomic mass units (amu). This measurement is found by totaling the molar mass for each element. For example, the molar mass of Aluminum chloride ($AlCl_3$) is $M(AlCl_3) = (1 \times 26.9815\ amu) + (3 \times 35.453\ amu) = 133.3405\ amu$.

   Suppose we know that the following atomic masses of several compounds:

   Acetone: $M(C_3H_6O) = 58.080 amu$
   Acetaldehyde: $M(C_2H_4O) = 44.053\ amu$
   Citric Acid: $M(C_6H_8O_7) = 192.12\ amu$

   Use this information to find the atomic mass of water (H_2O). (*Hint: you will need to use a system of equations and matrices to find the necessary individual atomic masses of C, H, and O … not Google!*) When you have an answer, check your answer online.