5C-2 Assignment

1. Caleb rows a boat upstream in a river for 3 miles in 90 minutes. He then turns around and rows the same distance downstream in only 1 hour. If both the rowing speed and current speed are constant, find Caleb's rowing speed and the speed of the current.

$$r = speed\ of\ row\ boat\ in\ still\ water, \qquad c = speed\ of\ current$$

Up-river trip: $3 = (r - c)1.5$

Down-river trip: $3 = (r + c)1$

$$\begin{cases} 3 = (r-c)1.5 \\ 3 = (r+c)1 \end{cases} = \begin{cases} 3 = 1.5r - 1.5c \\ 3 = r + c \end{cases} = \begin{cases} 6 = 3r - 3c \\ 9 = 3r + 3c \end{cases}$$

Adding the equations: $\mathbf{15 = 6r \rightarrow r = \frac{15}{6} = 2.5mph}$

Subtracting the equations: $\mathbf{-3 = -6c \rightarrow c = \frac{3}{6} = .5\ mph}$

$$\boldsymbol{rowing\ speed = 2.5mph, \qquad current\ speed = .5mph}$$

2. An airplane travels 45 minutes into a headwind (against the wind) for 100 miles. After dropping off some passengers, it travels the same distance with the same wind that is now a tailwind (the wind pushes the plane from behind) in only 20 minutes. If both the wind speeds and the airplane speeds are constant, find the speed of the airplane (in still air) and the speed of the wind.

$$a = airplane\ speed\ with\ no\ wind, \qquad w = speed\ of\ wind$$

Into wind: $100 = (a - w)45$

With wind: $100 = (a + w)20$

$$\begin{cases} 100 = (a-w)45 \\ 100 = (a+w)20 \end{cases} = \begin{cases} 100 = 45a - 45w \\ 100 = 20a + 20w \end{cases} = \begin{cases} 200 = 90a - 90w \\ 450 = 90a + 90w \end{cases}$$

Adding the equations: $650 = 180a \rightarrow a = \frac{650}{180} = \mathbf{3.611mpm = 216.67mph}$

Subtracting the equations: $250 = 180w \rightarrow w = \frac{250}{180} = \mathbf{1.38mpm = 83.33mph}$

$$\boldsymbol{airplane\ speed = 216.67mph, \qquad wind\ speed = 83.33mph}$$

3. Suppose we have two large bottles of hydrochloric acid solution. Bottle A is 5% HCl and bottle B is 35% HCl. How much of each solution do we need to acquire a 50 ml of 15% HCl?

$$a = volume\ of\ 5\%\ HCL\ needed, \qquad b = volume\ of\ 35\%\ HCL\ needed$$

Volume equation: $a + b = 50$

Concentration equation: $.05a + .35b = .15(50)$

$$\begin{cases} a + b = 50 \\ .05a + .35b = .15(50) \end{cases} = \begin{cases} a + b = 50 \\ a + 7b = 150 \end{cases}$$

Subtracting equations: $6b = 100 \rightarrow b = \frac{100}{6} = \frac{50}{3}$
Substituting: $a + \frac{50}{3} = 50 \rightarrow a = \frac{100}{3}$

$$\boldsymbol{We\ need\ \frac{100}{3}\ ml\ of\ solution\ A, and\ \frac{50}{3}\ ml\ of\ solution\ B}$$

4. Suppose we have two large bottles of saline solution (a mixture of water, H_2O, and sodium chloride, $NaCl$). Bottle A is 10% $NaCl$, and bottle B is 20% $NaCl$. How much of each solution do we need to acquire a 10 ml of 13% $NaCl$?

$$a = amount\ of\ 10\%\ NaCl, \qquad b = amount\ of\ 20\%\ NaCl$$

Volume Equation: $a + b = 10$
Concentration Equation: $.10a + .20b = .13(10)$

$$a + b = 10$$
$$.10a + .20b = .13(10)$$
$$(b = 3, a = 7)$$
$$\boldsymbol{We\ need\ 3ml\ of\ 10\%NaCl, and\ 7ml\ of\ 20\%\ NaCl}$$

5. **Note: Original problem had faulty data! The correct numbers are in bold below…**
The molar mass of a compound is the mass of one mole of that compound measured in atomic mass units (amu). This measurement is found by totaling the molar mass for each element. For example, the molar mass of Aluminum chloride ($AlCl_3$) is $M(AlCl_3) = (1 \times 26.9815\ amu) + (1 \times 106.359\ amu) = 133.3405\ amu$.

Suppose we know that the following atomic masses of several compounds:
Acetone: $M(C_3H_6O) = 17.031\ amu$ **(58.08)**
Acetaldehyde: $M(C_2H_4O) = 59.067\ amu$ **(44.053)**
Citric Acid: $M(C_6H_8O_7) = 80.043\ amu$ **(192.12)**
Use this information to find the atomic mass of water (H_2O). (*Hint: you will need to use a system of equations and matrices to find the necessary atomic masses… not Google!*) Check your answer online.
Let $c = mass\ of\ Carbon,\ h = mass\ of\ Hydrogen,\ x = mass\ of\ Oxygen$

$$3c + 6h + x = 17.031$$
$$2c + 4h + x = 59.067$$
$$6c + 8h + 7x = 80.043$$

$\left(x = \frac{143139}{1000}, h = \frac{334857}{2000}, c = -\frac{376893}{1000}\right)$Solve the system! Using matrices is a good technique.

$$\begin{bmatrix} 3 & 6 & 1 & | & 17.031 \\ 2 & 4 & 1 & | & 59.067 \\ 6 & 8 & 7 & | & 80.043 \end{bmatrix} \rightarrow (c = -376.89, h = 167.43, x = 143.139)\ These\ are\ wrong\ ...why?$$

Correct system:

$$\boldsymbol{3c + 6h + x = 58.08}$$

$$\mathbf{2c + 4h + x = 44.053}$$
$$\mathbf{6c + 8h + 7x = 192.12}$$

$$\left[\begin{array}{ccc|c} 3 & 6 & 1 & 58.08 \\ 2 & 4 & 1 & 44.053 \\ 6 & 8 & 7 & 192.12 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12.010 \\ 0 & 1 & 0 & 1.009 \\ 0 & 0 & 1 & 15.999 \end{array}\right] \rightarrow c = 12.010, h = 1.009, x = 15.999$$

So, the mass of $\boldsymbol{H_2O}$ is $\mathbf{2(1.009) + (15.999) = 18.017}\,\boldsymbol{g/mol}$