5B: Solving 3-Variable Linear Systems

From Two Variables to Three

We have now seen that if we have two equations with the same two variables x and y, we can consider this a system of two equations and solve for the ordered pair (x, y) that is a solution of both equations. (Note that we must have two equations to solve for a unique solution.)

Geometrically, we found that the solution is the point of intersection of the two lines. This also shows that we can have 0, 1, or infinite solutions depending on where (or if) they intersect.

Example Solve this system by **substitution**

$$\begin{aligned} 2x - y + z &= 0, \qquad E_1 \\ x + 2y + 4z &= 4, \qquad E_2 \\ z &= x - y, \qquad E_3 \end{aligned}$$

Step 1: Substitute E_3 into E_1 and E_2 to write two new equations in terms of x and y.

Step 2: Solve these two equations for x and y.

Step 3: Use these values to solve for z.

Example Solve this system using Gaussian Elimination

$$\begin{aligned} &2x - y + z = 0 \\ &x + 2y + 4z = 4 \\ &-x + y + z = 0 \end{aligned}$$

$$(x = 2, y = 3, z = -1)$$

Example Solve this system using matrices and elementary row operations.

$$\begin{aligned} &2x - y + z = 0 \\ &x + 2y + 4z = 4 \\ &-x + y + z = 0 \end{aligned}$$

$$(x = 2, y = 3, z = -1)$$

Example Solve this system using matrices and elementary row operations.

$$\begin{aligned} &x + y + z = 10 \\ &2x - 3y + z = 5 \\ &2x + 2y + 2z = 20 \end{aligned}$$

$$\left(x = -\frac{4\,z}{5} + 7, y = -\frac{z}{5} + 3, z \in \mathbb{R}\right)$$

Exercises

1. Solve this system by substitution

$$\begin{aligned} 3x - y + 2z &= -2 \\ y + 3z &= 3 \\ 2z &= 4 \end{aligned}$$

$$(x = -3, y = -3, z = 2)$$

2. Solve using Gaussian Elimination.

$$\begin{aligned} x - y + z &= 0 \\ 2x - 3z &= -1 \\ -x - y + 2z &= -1 \end{aligned}$$

$$(x = 1, y = 2, z = 1)$$

3. Solve the system using matrices and elementary row operations

$$\begin{aligned} 2x + 3y - 2z &= 2 \\ 3x + 3y - z &= 6 \\ -3y + 3z &= 3 \end{aligned}$$

$$(x = 1, y = 2, z = 3)$$

4. Solve the system using any method.

$$\begin{aligned} x + 2y &= 4 \\ 3x + 4y &= 5 \\ 2y + z &= 8 \end{aligned}$$

$$\left(x = -3, y = \frac{7}{2}, z = 1\right)$$