5A-2: Intro. To Matrices With Systems

From Equations to Matrices

For most applications in math and science, systems of linear equations are written in standard form and solved using of a powerful tool called matrix algebra.

Explore: From Equations to Matrices

Solve this system by elimination. After each step, write *both* equations in the *Equation Form* column, record your steps in the *Step Description* column. (Just do the first 2 columns for now.)

Equation Form	Step Description *(record changes)*	Matrix Form	Row Operations *(record changes)*
$3x + 2y = 6$ $-4x - 3y = -7$	E_1 E_2	$\left[\begin{array}{cc|c} 3 & 2 & 6 \\ -4 & -3 & 7 \end{array}\right]$	R_1 R_2

<u>*The question is:* *“What is a matrix?”*</u>

A **matrix** is a rectangular array of numbers that can be manipulated algebraically. For example, if we have the system of linear equations

$$\begin{aligned} 3x + 2y &= 6 \\ -4x - 3y &= -7, \end{aligned}$$

we can write the coefficients as the matrix

$$\mathbf{A} = rows\left\{\underbrace{\begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix}}_{columns}\right..$$

The **dimension** of a matrix is given as $rows \times columns$. So, the matrix above is a 2×2 matrix and the constants in the system above make the 2×1 matrix

$$\mathbf{b} = \begin{bmatrix} 6 \\ -7 \end{bmatrix}.$$

When working with a system of equations, we often like to write the system as an **augmented matrix** that contains the coefficients from the left side of the equations and the constants from the right side. The following is an *augmented* matrix for the system above

$$[\mathbf{A}|\mathbf{b}] = \left[\begin{array}{cc|c} 3 & 2 & 6 \\ -4 & -3 & 7 \end{array}\right]$$

- ***<u>Explore</u>***: Now go back to the exploration above and write the augmented matrix for each step and the description of the row changes.

Solving Linear Systems with Matrices

As we see in the explorations, matrices are a nice way to organize the information needed to solve a system. Our goal when working with matrices to solve a system is to change the matrix into **reduced row echelon form (*rref*)** which means that we have only one 1 in each column of the the coefficient and 0’s in the rest of the column. We also want the first 1 to be in the first row, the next 1 to be in the 2nd row, etc. A 2×2 matrix in *reduced row echelon form* looks like this:

$$\left[\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & * \end{array}\right]$$

where the * represents any number.

Below are the steps we can use to change a matrix into row echelon form.

<u>Elementary Row Operations:</u>

When changing a matrix into *row echelon form*, we are may use any of the following operations:

1. Interchange two rows.
2. Multiply a row by a nonzero number (called a “scalar”).
3. Add a constant multiple of one row to another.

Exercises

For each system of equations below, write as a matrix and use elementary row operations to change into reduced row echelon form and solve the system. (one has no solution, and one has infinite solutions.)

1. $2x + y = 10$
   $x - 2y = -5$

$$(x = 3, y = 4)$$

2. $2x - 4y = 8$
   $-6x + 8y = -32$

$$(x = 8, y = 2)$$

3. $2x - 3y = -23$

$$x + y =$$

$$\left(x = -\frac{23}{5}, y = \frac{23}{5}\right)$$

4. $2x - 4y = 8$
   $-x + 2y = 4$

$$No\ Solution$$

5. $2x - 3y = 5$
   $-6x + 9y = -15$

$$Solution: 2x - 3y = 5$$

Matrices on your TI-8x calculator.

You can use your TI-8x graphing calculator to find the reduced row echelon form of an augmented matrix with the following steps:

i. Go to the Matrix menu by hitting [2nd], [MATRX].
ii. Go to EDIT and choose [A] (or some other matrix name), change the dimensions to 2 x 3, and type in the values (like figure 1 below)
iii. Go to the Matrix menu, choose MATH, and select "rref(" (like figure 2 below)
iv. Now go back to the Matrix menu and select the matrix name [A], then press enter to get the reduced row echelon form (like figure 3 below).

__Example__ Use your graphing calculator to solve the system to the 3 decimal places.

$$3x + 4y = 20$$
$$5x - 7y = 32$$

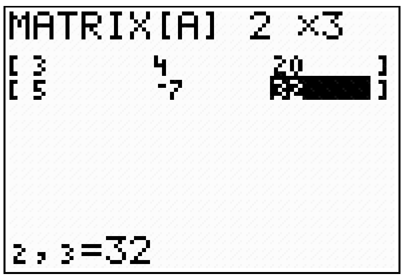

figure 1

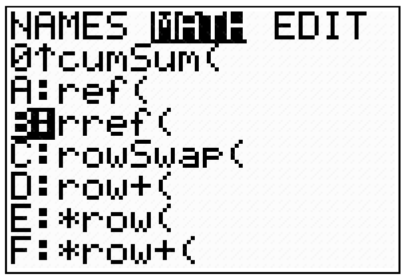

figure 2

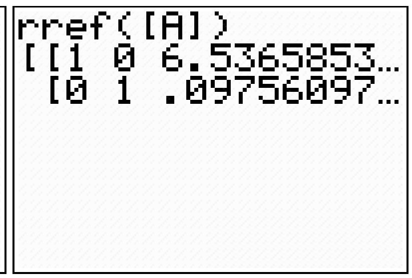

figure 3

$$\left[\begin{array}{cc|c} 3 & 4 & 20 \\ 5 & -7 & 32 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 6.537 \\ 0 & 1 & .097 \end{array}\right] \rightarrow \ x = 6.537, y = .097$$

Exercises (continued)

For each system below,

a) write the augmented matrix that corresponds to the system,
b) then use your Graphing calculator to find the reduced row echelon form (copy this down),
c) then state your solution to three decimal places.

6. $5x - 7y = -9$
   $-3x + y = -1$

$$(x = 1, y = 2)$$

7. $7x - 12y = 52$
   $4x + 8y = -10$

$$\left(x = \frac{37}{13}, y = -\frac{139}{52}\right)$$

8. $3x + 6y = 9$
$-15x - 30y = -45$

$Solution: 3x + 6y = 9$