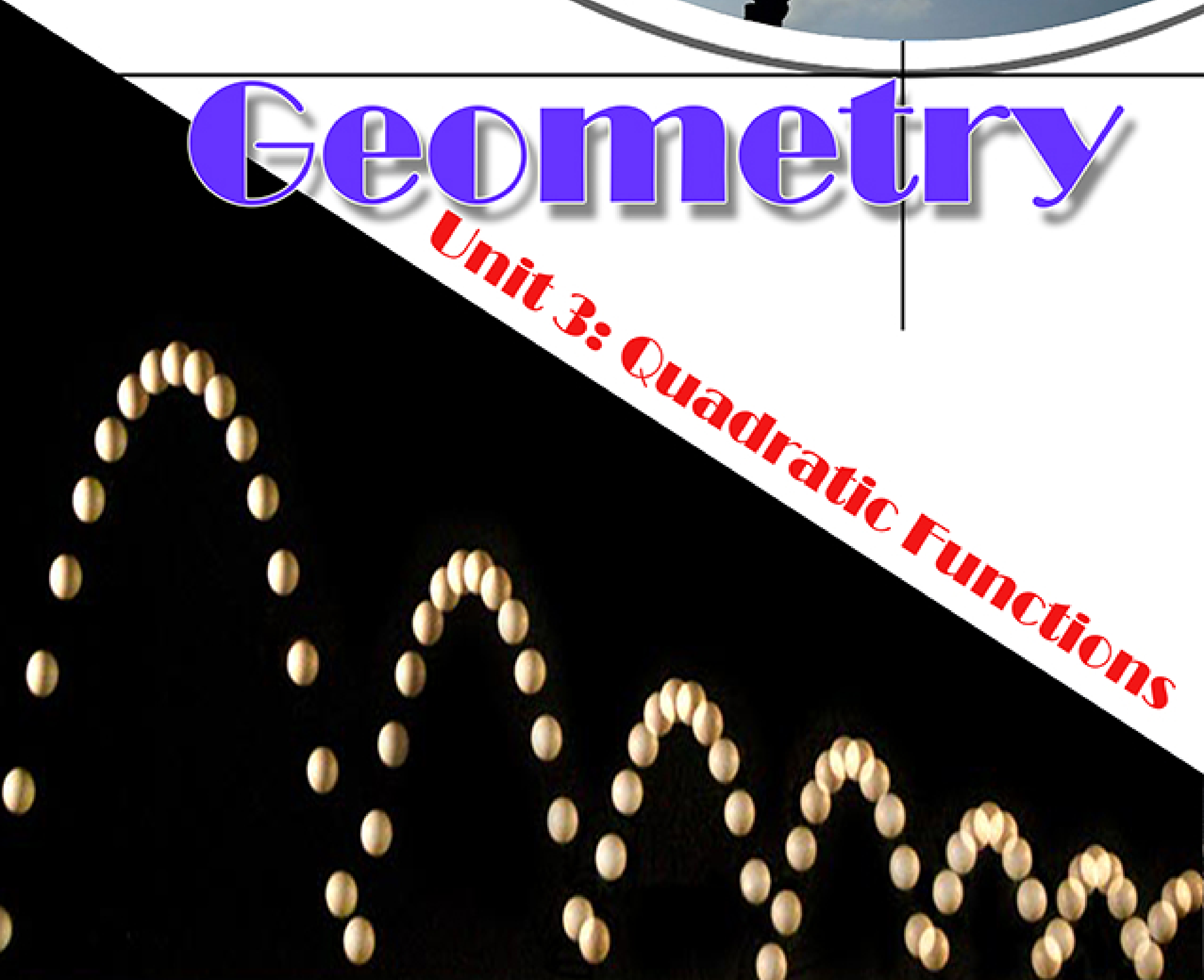
Geometry

Unit 3: Quadratic Functions



Warmups and Startups

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Warmups

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Class Notes and Activities

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Starburst Growth

A sequence of Starbursts is shown below.

Figure 1

Figure 2

Figure 3

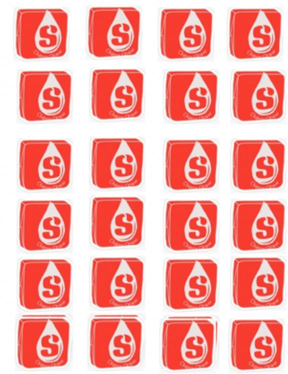


Figure 4

1. Describe what is changing from one figure to the next.

2. What would Figure 5 look like? Can you determine the number of Starbursts in Figure 5 without counting them one by one?

3. a. Complete the table with the number of Starbursts in each figure. Show the calculation you used to figure it out.

Figure Number	# of Starbursts
1	
2	
3	
4	
5	
10	

   b. Is the number of Starbursts increasing by the same amount every time? Explain.

   c. Can the number of Starbursts in each figure be described as an arithmetic sequence? A geometric sequence? Explain.

4. Let $f(n)$ be the number of Starbursts in the nth figure. Write an equation for $f(n)$.

Lesson 7.1 – Quadratic Growth

QuickNotes

Check Your Understanding

1. Cindy is doing a 30-day jumping jack challenge. On the first day she does 5 jumping jacks. On the second day, she does 10 jumping jacks. On the third day, she does 15 jumping jacks. This pattern continues.
   a. Let $j(n)$ represent the number of jumping jacks Cindy does on the nth day of the challenge. Is $j(n)$ a linear function, a quadratic function, or neither? Explain.

   b. Let $t(n)$ represent the total number of jumping jacks Cindy has done by the nth day of the challenge. Is $t(n)$ a linear function, a quadratic function, or neither? Explain.

2. Let $g(x) = (x + 5)(2x - 3)$.
   a. Complete the table of values.

x	$g(x)$
-2	
-1	
0	
1	
2	

   b. Re-write $g(x)$ by multiplying the two factors.

	x +	5
2x		
-3		

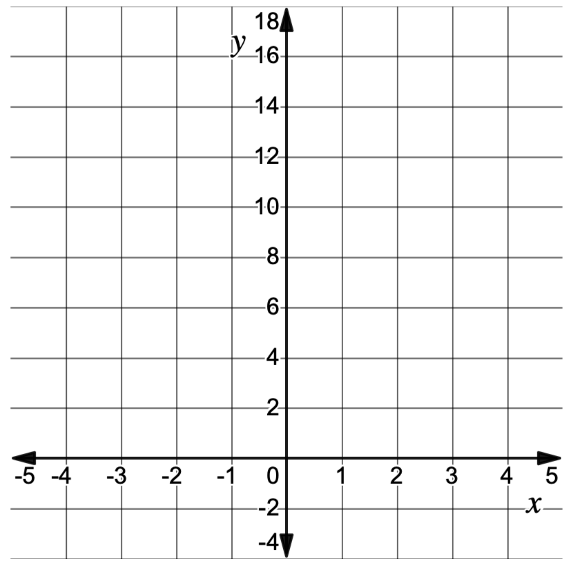
   c. Give at least two clues that indicate why $g(x)$ is a quadratic function.

Show Me a Graph!

Today we'll look at the most basic quadratic function $f(x) = x^2$.

1. Complete the table of values and plot the points.

x	$f(x)$
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



2. Connect the points with a smooth curve. What shape is made by the points?

3. What is the lowest point on the graph?

4. What is the domain of f? What is the range of f?

5. Where is f decreasing? Where is f increasing?

6. Does $f(x)$ have a constant rate of change? How can you tell from the graph?

7. a. Use a dashed line to draw the line of symmetry on the graph.

   b. Give two x-values that have the same output. What do you notice about the location of these x-values in relation to the line of symmetry?

Lesson 7.2 – The Quadratic Parent Function

QuickNotes

Check Your Understanding

1. For $f(x) = x^2$, $f(a) = 10$.
    a. Find the value of a.

    b. Find $f(-a)$.

2. Let $f(x) = x^2$.
    a. Which is greater: $f(7)$ or $f(-7)$? Explain.

    b. Which is greater $f(-10)$ or $f(3)$? Explain.

3. Does $f(x) = x^2$ change faster between $x = -4$ and $x = -3$ or between $x = -3$ and $x = -2$? Explain.

4. What is the constant second difference of $f(x) = x^2$?

Parabolas on the Move

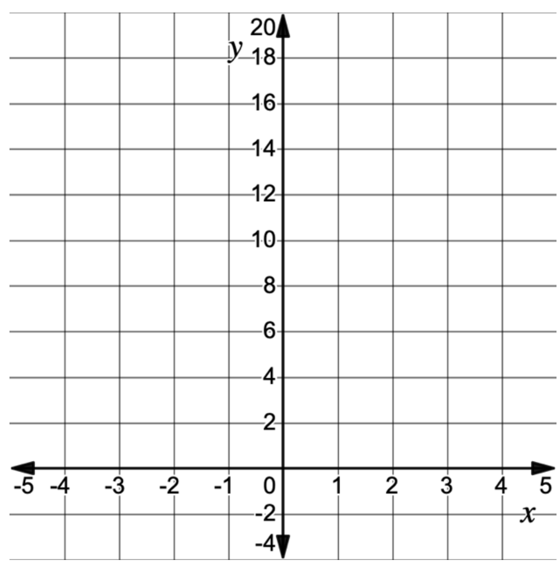
Yesterday we looked at the graph of the parent function $f(x) = x^2$. Today we'll see how making changes to this equation affects the graph.

1. Complete the table of values for $f(x) = x^2$ and $g(x) = x^2 + 3$.

x	$f(x)$	$g(x)$
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		

2. How do the values of g and f compare?

2. a. Graph $y = g(x)$ on the coordinate plane shown.
   b. How does the graph of f compare to the graph of g?



   c. What is the vertex of g?

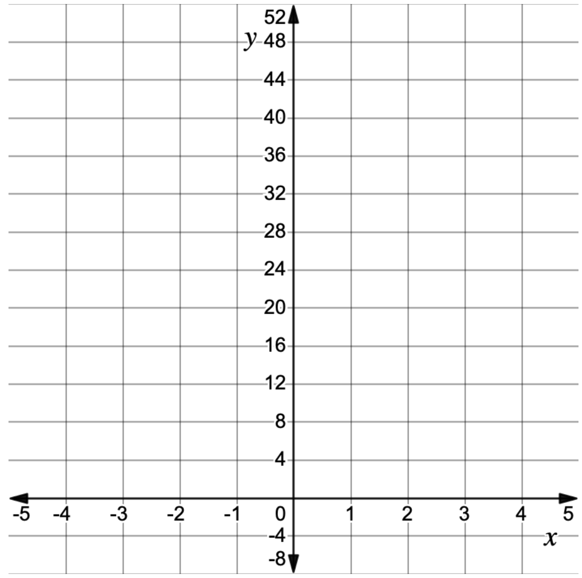
   d. What is the axis of symmetry of g?

   e. Identify the domain and range of g.

3. Predict what you think would happen if we graphed the function $y = x^2 - 3$.

4. Complete the table of values and graph $h(x) = 2x^2$.

x	$h(x)$
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



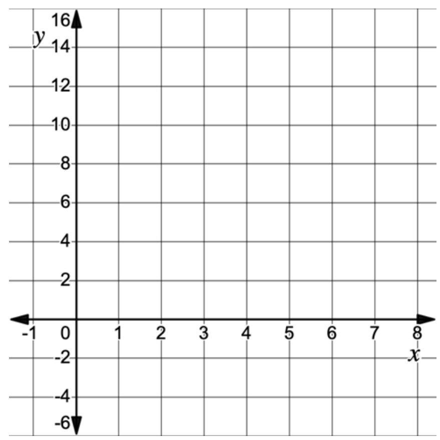
   a. Identify the vertex and axis of symmetry.

   b. What is the domain and range of h?

   c. Explain how the coefficient of “2” affected the graph.

5. Go to desmos.com and click “Graphing Calculator”.
   a. Graph $j(x) = (x-4)^2$ and sketch it to the right.
   a. Identify the vertex and axis of symmetry.

   b. What is the domain and range of j?



6. Let’s think about the function $k(x) = (x+3)^2$.
   a. Without graphing, can you predict the vertex and axis of symmetry of this parabola?

   b. Can you predict the domain and range of k?

   c. Graph $k(x)$ using Desmos to see if you are correct.

7. Challenge! Consider the function $f(x) = (x-4)^2 + 3$.
   a. Where is the vertex of this parabola? How do you know?

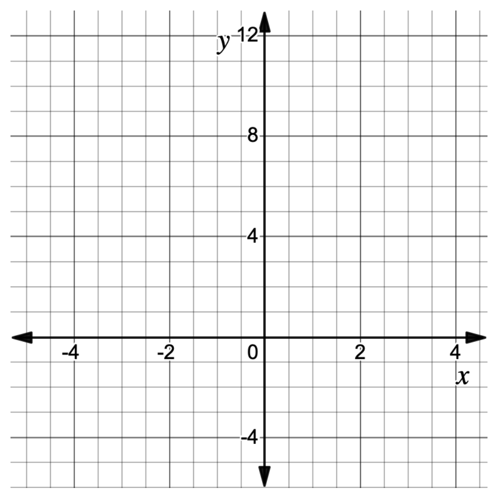
   b. Graph $f(x)$ using Desmos to see if you are correct.

Lesson 7.3 – Transforming Quadratic Functions

QuickNotes

Check Your Understanding

1. Let $g(x) = (x + 2)^2 - 4$.
   a. Identify the transformations that occurred from the parent function $p(x) = x^2$.

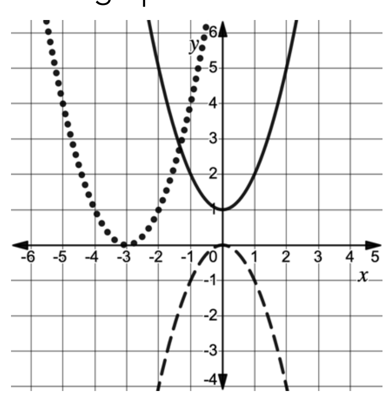


   b. Identify the vertex of the parabola.

   c. Graph $y = g(x)$.

2. Three parabolas and their equations are given. Label each graph with its correct function name.

   $f(x) = -x^2$
   $g(x) = x^2 + 1$
   $k(x) = (x + 3)^2$

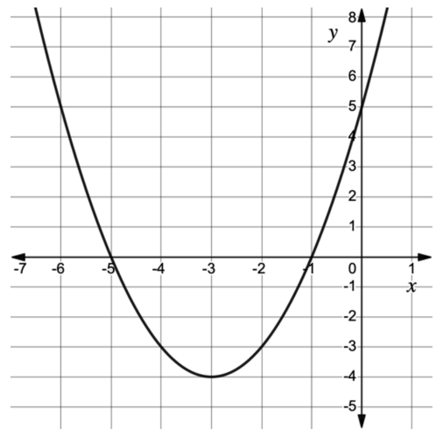


3. Explain how you can tell if a parabola will open up or open down.

Parabola Patterns

We've already seen that quadratic functions follow particular patterns. Today we'll look at some patterns made by their graphs.

1. The graph of a quadratic function $y = f(x)$ is shown.



   a. Identify the axis of symmetry.

   b. Identify the x-intercepts.

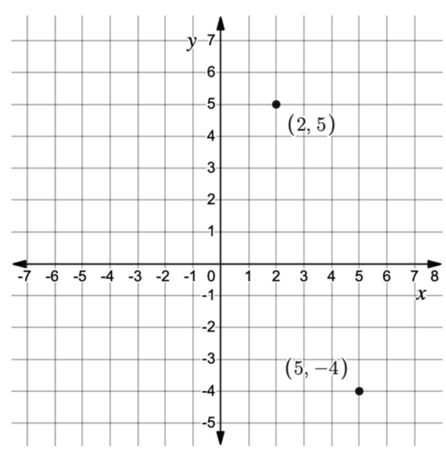
   c. What is the relationship between the location of the x-intercepts and the axis of symmetry?

2. a. For which x-values is $f(x) = 5$?

   b. Show these values on the graph. What is the relationship between these x-values and the axis of symmetry?

3. If $f(4) = 45$, which other x-value gives an output of 45? Explain.

4. A parabola has a vertex at (2,5) and passes through the point (5, -4).



   a. Does this parabola open up or down? How do you know?

   b. Give one other point that you know for sure is on this parabola.

   c. The parabola has an x-intercept at $x = 4.236$. Find the other x-intercept.

Lesson 7.4 – Features of Quadratic Functions

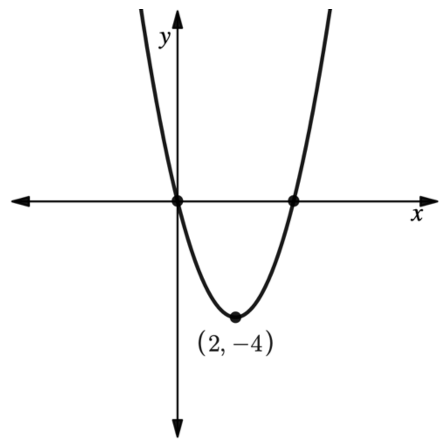
QuickNotes

Check Your Understanding

1. The graph of a quadratic function f is shown.
   a. Identify the axis of symmetry.

   b. Find both x-intercepts.

   c. Is $f(5) = f(-5)$? Explain.



2. A table of selected values is given for a quadratic function g.

x	-11	-10	-9	-8	-7	-6	-5	-4
$g(x)$	7	0	-5	-8	-9	-8	-5	0

a. Identify both x-intercepts of the graph of g.

b. Find the vertex and axis of symmetry of the graph of g.

c. Does this parabola open up or down? How do you know?

d. Find $g(-3)$.

Quadratics in Disguise

Today we'll look at the function $f(x) = (x-4)(x+2)$.

1. Explain how you know that f is a quadratic function.

2. Find $f(0)$. What does this value tell you about the graph of f?

3. a. Find $f(4)$ and $f(-2)$.

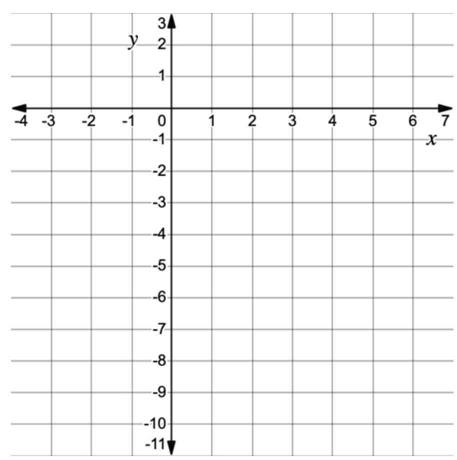
   b. What do these values tell you about the graph of f?

   c. How do these values relate to the original equation?

4. What is the axis of symmetry of the graph of f? How do you know?

5. Find the vertex of f. Is the vertex a maximum or a minimum? Explain.

6. Use your work above to sketch the graph of $y = f(x)$.



7. Write an equation for $f(x)$ in vertex form.

8. Sasha says that $f(x)$ can also be written as $f(x) = x^2 - 2x - 8$. Do you agree? Explain.

9. Find $f(0)$ using Sasha's equation. What do you notice?

Lesson 7.5 – Forms of Quadratic Functions

QuickNotes

Check Your Understanding

1. Let $f(x) = -3(x+1)(x-7)$ and $g(x) = 3x^2 - 12x + 20$.
    a. Find the x-intercepts of $f(x)$.
    b. Find the vertex of g.
    c. Which function has the greater y-intercept?

2. Let $f(x) = (x-8)(x-2)$ and $g(x) = (x-5)^2 - 9$. Prove that $f(x) = g(x)$.

3. A parabola can be written in vertex form as $y = (x-6)^2 - 9$ and in intercept form as $y = (x-3)(x-k)$ for some constant k.
    a. Find the axis of symmetry of this parabola.
    b. Identify one of the x-intercepts.
    c. Find the value of k.

Factor That!

> We saw earlier this chapter that a quadratic written in factored form can be converted to standard form by multiplying out the two factors. Can we do this in reverse? Today we'll see how to write a quadratic in standard form into factored form.

1. Write $f(x) = (x+5)(x+2)$ in standard form by multiplying the two factors.

	x	5
x		
2		

2. Write $g(x) = (x-4)(x+7)$ in standard form by multiplying the two factors.

3. What relationships do you notice between the equation of a quadratic in factored form and the equation of the same quadratic in standard form?

4. Let's see if we can do this in reverse. Fill in the blanks with the missing numbers.

   a. $x^2 + 8x + 15 = (x+3)(x + \_\_\_)$

   b. $x^2 + 5x + 6 = (x + \_\_\_\_)(x + 2)$

   c. $x^2 + 11x + 24 = (x + \_\_\_\_)(x + \_\_\_)$

   d. $x^2 - 11x + 24 = (x - \_\_\_\_)(x - \_\_\_\_)$

   e. $x^2 - 7x + 6 = (x - \_\_\_\_)(x - \_\_\_\_)$

   f. $x^2 + 6x - 16 = (x+8)(x - \_\_\_\_\_)$

   g. $x^2 + 3x - 4 = (x + \_\_\_\_)(x - \_\_\_\_\_)$

   h. $x^2 - 3x - 4 = (x + \_\_\_\_)(x - \_\_\_\_\_)$

5. What patterns do you notice?

6. Are you ready for more?

   a. $3(x^2 + 6x + 5) = 3(x + \_\_\_\_\_)(x + \_\_\_\_\_)$

   b. $3x^2 + 18x + 15 = \_\_\_(x + \_\_\_\_\_\_)(x + \_\_\_\_\_)$

Lesson 7.6 – Writing Quadratics in Factored Form

QuickNotes

Check Your Understanding

1. Are $f(x) = x^2 - 7x - 44$ and $g(x) = (x + 11)(x - 4)$ equivalent functions? Explain.

2. Write each quadratic expression in factored form.
   a. $x^2 + 13x + 36$

   b. $x^2 - 7x + 10$

   c. $x^2 - 9x - 8$

3. Consider the function $f(x) = 2x^2 + 4x - 48$ written in standard form.
   a. How is this function different than the ones you saw in question 2?

   b. Re-write $f(x)$ by factoring out the greatest common factor.

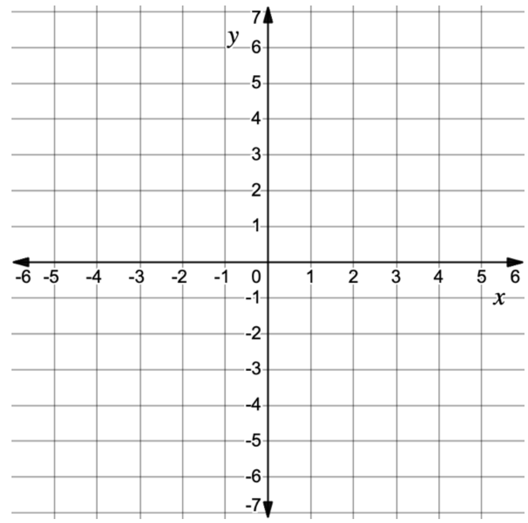
      $f(x) =$ ____( )

   c. Factor $f(x)$ completely.
      $f(x) =$ ____( )( )

Solving Quadratic Puzzles

Yesterday we learned how to write quadratics in intercept form by finding its factors. Today we'll explore how using factored form can help us solve quadratic equations.

1. Graph $f(x) = (x-2)(x+3)$ using Desmos. Then sketch it below.



   a. What is the y-intercept of the graph of f? How do you know?

   b. What are the x-intercepts of the graph of f?

   c. When is $f(x) = 0$?

   d. How do the intercepts relate to the equation?

2. Let $g(x) = (x-1)(x-4)$.
   a. Without graphing, determine the y-intercept of the graph of g.

   b. Without graphing, determine the x-intercepts of the graph of g.

   c. When is $g(x) = 0$?

   d. Check your work by graphing the function in Desmos.

3. What is the relationship between the factored form of a quadratic equation, the x-intercepts, and the values at which the function is 0?

4. We'll explore this relationship further with some number puzzles.
    a. I'm thinking of two numbers that multiply to 24. What could my numbers be?

    b. I'm thinking of two numbers that multiply to 15. What could my numbers be?

    c. I'm thinking of two numbers that multiply to 0. What could my numbers be?

5. Here are a few more puzzles.
    a. What value(s) of ♥ would make 7(♥)=0?
    b. What value(s) of *a* would make (a)(-3.25)= 0?
    c. What value(s) of ♥ would make 0(♥)=0?
    d. What value(s) of a would make (100)(5-a)=0?
    e. What value(s) of x would make (x-9)(23)=0?
    f. What value(s) of x would make (x-4)(x+2)=0?

6. a. What values of x would make $(x + 5)(x + 3)$=0?

   b. Find the x-intercepts of $f(x) = (x + 5)(x + 3)$.

7. What values of x would make $x^2 + 8x + 15 = 0$?

8. Find the x-intercepts of $h(x) = x^2 - 3x - 10$.

Lesson 7.7 – Solving Quadratics with the Zero Product Property

QuickNotes

Check Your Understanding

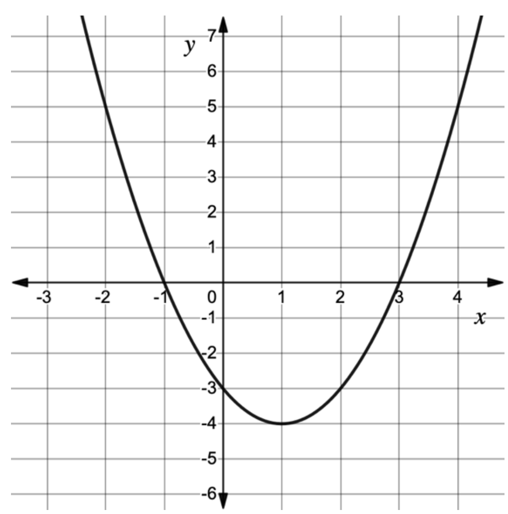
1. Let $f(x) = (x-3)(x-10)$. When is $f(x) = 0$?

2. Find the x-intercepts of $g(x) = x^2 - 12x + 32$.

3. Solve $x^2 - 6x - 27 = 0$.

4. The graph of h is shown. Which of the following equation(s) represent $h(x)$? Choose all that apply.

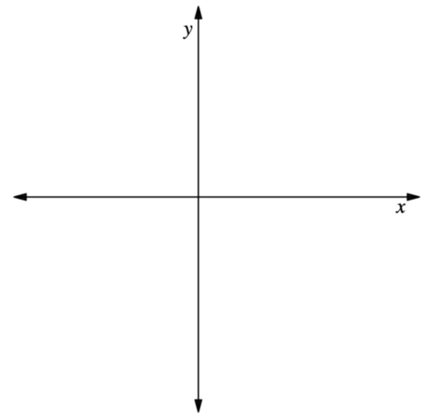
   A) $h(x) = (x-1)(x+3)$
   B) $h(x) = (x-1)^2 - 4$
   C) $h(x) = x^2 - 2x - 3$
   D) $h(x) = (x+1)^2 - 4$
   E) $h(x) = (x-3)(x+1)$
   F) $h(x) = x^2 + 2x - 3$



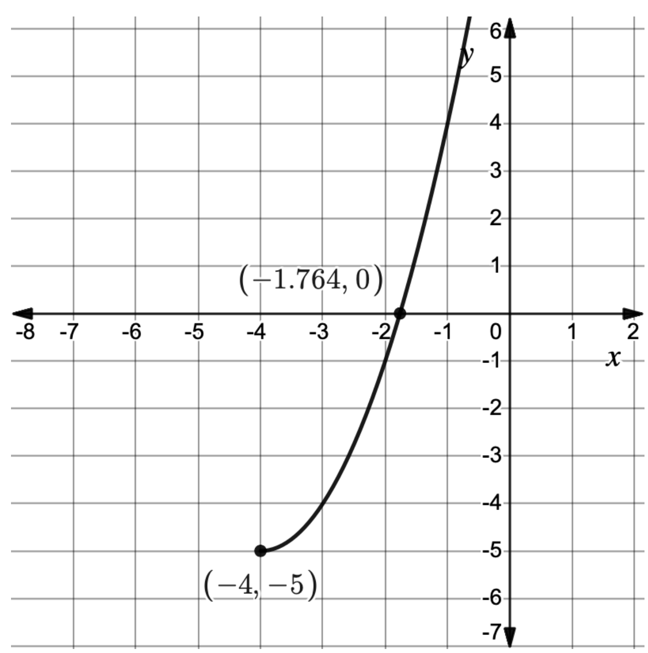
How Do We Find Zeros?

We've seen that we can find the zeros of a quadratic by looking for x-intercepts on a graph, or by rewriting the equation in factored form. But some quadratics are not easily factored. Today we'll look at another strategy for finding zeros.

1. A parabola has a vertex at (2, -5). The horizontal distance between the vertex and an x-intercept is 4.
   a. Draw a rough sketch of this scenario.
   b. Find both x-intercepts. Clearly demonstrate your strategy.



2. Janyce was given an equation for a parabola. Unfortunately, her calculator is broken, and it only shows half the parabola, as seen below. Can you help her figure out the missing information?
   a. What is the x-coordinate of the vertex?
   b. What is the horizontal distance between the vertex and the x-intercept?
   c. Find the other x-intercept.



3. The equation Janyce was trying to graph was $f(x) = x^2 + 8x + 11$. Let's see how we can use this equation to identify the intercepts even if we don't have a graph.
   a. Remember that the standard form of a quadratic is $ax^2 + bx + c$. Identify a, b, and c, for this parabola.

   b. Calculate the x-coordinate of the vertex. Let's call this value *h*.

   c. Calculate $\sqrt{h^2 - \frac{c}{a}}$. What does this value represent?

   d. Explain how you could find both zeros of this function using your work above.

4. For each of the quadratics, find the vertex and the horizontal distance between the vertex and an x-intercept. Then find both x-intercepts.

Equation	Vertex at x=______	Horizontal distance between vertex and x-intercept	First x-intercept	Second x-intercept
$y = 2x^2 + 2x - 17.5$				
$y = x^2 - 6x + 3$				
$y = 3x^2 + 24x + 48$				

Lesson 7.8 – Solving Quadratics using Symmetry

QuickNotes

Check Your Understanding

1. Find the x-intercepts of $f(x) = x^2 - 4x - 21$.

2. Let $g(x) = 3x^2 + 19x - 14$.
   a. Identify the axis of symmetry.
   b. What is the horizontal distance between the vertex and each x-intercept?
   c. Find both x-intercepts.

3. When is $2x^2 + 5x + 1 = 0$?

How Much Revenue Can a Waterpark Make?

A waterpark sells daily admission tickets. Their revenue (total sales) is determined by the number of tickets they sell and the price per ticket.

1. How do you think changing the price of the ticket will affect the waterpark's revenue?

2. A waterpark currently sells daily admission tickets for \$40 and typically sells 500 tickets each day. How much revenue does the park make in a single day?

3. The waterpark wants to increase their revenue, so they are thinking about increasing their price. A survey found that for each \$1 increase in price, 10 fewer tickets are sold.
   a. If the price of a ticket were \$42, how many tickets would they sell? How much revenue would the waterpark make?

   b. If the price of a ticket was \$55, how much revenue would the waterpark make?

   c. Is increasing the price of the ticket always a good idea? Why or why not?

4. The waterpark's revenue when selling tickets at a price of p dollars can be modeled by the function $r(p) = p(900 - 10p)$.
   a. Find $r(40)$ and interpret this value in the context of this problem.

   b. At which price will the waterpark make no revenue? How do you know?

   c. How much should the waterpark charge per ticket to maximize the revenue? Explain.

   d. What is the maximum revenue?

5. At which price will the waterpark make a daily revenue of \$18,000?

Lesson 7.9 – Quadratic Models

QuickNotes

Check Your Understanding

1. A tourist climbed to the top of a lighthouse and dropped her sunglasses. The height of the sunglasses, in feet, can be modeled by the function $H(t) = 108 - 16t^2$ where t is the time in seconds since the sunglasses were dropped.
   a. From what height were the sunglasses dropped? How do you know?

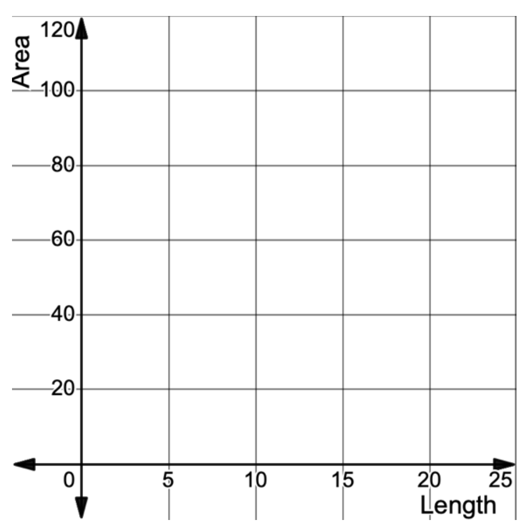
   b. After how many seconds will the sunglasses hit the ground?

2. A gardener has 40 feet of wire fencing to enclose a rectangular vegetable bed. Let l be the length of the vegetable bed.
   a. Write an expression in terms of l that represents the width of the vegetable bed.

   b. Write an equation for $A(l)$, the area of the vegetable bed with length l.

   c. Graph $A(l)$.

   d. If the gardener wants to maximize the area of the vegetable bed, what should be the length and width? Explain.



Quadratic Functions Unit Review

A. Find the range of the function $y = -(x+3)^2 - 7$.

B. Find the vertex of the function $f(x) = (x+10)(x+6)$.

C. Values of a quadratic function f are given.

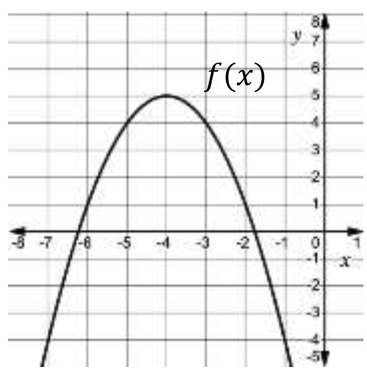
x	$f(x)$
-4	23
-1	5
0	3
2	5
3	9

a. Identify the axis of symmetry.
b. Find $f(5)$.

D. What are the zeros of $g(x) = (x+9)(2x-5)$?

E. Three quadratic functions are given below.

x	$g(x)$
-5	-1
-3	5
-1	7
1	5
3	-1



$h(x) = -3(x-2)^2 + 3$

Which function has the greatest maximum value? Explain.

F. Solve $3x^2 + 12x - 8 = 0$ algebraically.

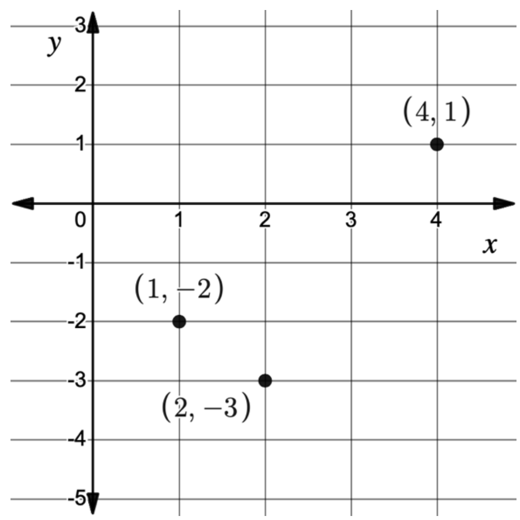
F. Solve $3x^2 + 12x - 8 = 0$ algebraically.

G. Write $g(x) = x^2 - 2x - 63$ in factored form.

H. The point $(2, -3)$ is the vertex of the graph of a quadratic function. Two other points on the graph of this quadratic are shown.

a. Plot two additional points that must be on the graph of this quadratic.

b. The equation of the quadratic can be written as $y = x^2 - 4x + c$. Find the value of c.



I. Describe the transformation that was performed on the parent function $p(x) = x^2$ to obtain the graph of $f(x) = 3x^2$.

J. The graph of the parent function $f(x) = x^2$ was shifted to the right 5 units and down 10 units. Write the equation of the resulting graph.

K. Consider the functions $f(x) = x^2 - 10x + 28$ and $g(x) = (x-5)^2 + 3$.

Which of the following statements is true?

A) $f(x)$ has a higher y-intercept than $g(x)$
B) $g(x)$ has a lower minimum value than $f(x)$
C) The graph of $f(x)$ opens up while the graph of $g(x)$ opens down
D) $f(x)$ and $g(x)$ have the same y-intercept and the same minimum value

L. Which of the following functions does NOT have x-intercepts at integer values?

A) $y = \frac{1}{2}(x+5)(x+8)$
B) $y = x^2 - 7x + 10$
C) $y = (x+2)(3x-8)$
D) $y = x^2 - 4$

M. Write the equation $y = (x+7)^2 - 13$ in standard form.

N. Which product is equivalent to $x^2 - 8x - 48$?

A) $(x-16)(x+3)$
B) $(x-12)(x+4)$
C) $(x-8)(x+4)$
D) $(x+12)(x-4)$

O. Find the minimum value of the graph of $f(x) = x^2 + 24x - 18$.

P. The length of a rectangular field is 7 feet longer than double the width of the field. If the width of the field is w, write an equation for $A(w)$, the area of the field with width w.

Q. A ball is thrown into the air from the top of a building. The height of the ball above the ground, in feet, t seconds after it is thrown, can be modeled by the function $h(t) = -16(t-2)^2 + 146$ where $h(t)$ is in feet.

a. From what height was the ball initially thrown?

b. After how many seconds does the ball reach its maximum height?

c. What is the maximum height of the ball?

d. After how many seconds will the ball reach the ground?

R. The first four figures of a visual pattern are shown below.

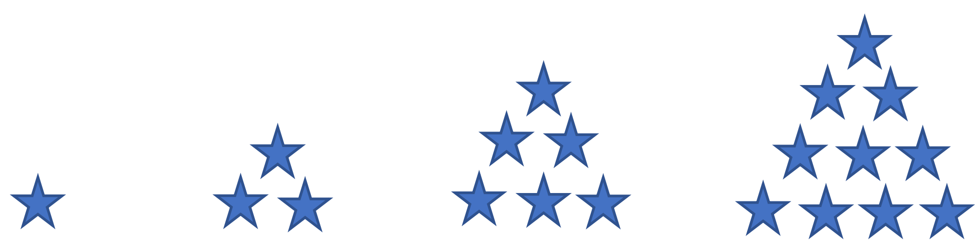


Figure 1 Figure 2 Figure 3 Figure 4

Let n be the figure number and $f(n)$ be the number of stars in Figure n. Is $f(n)$ a linear function, a quadratic function, an exponential function, or neither? Explain.

S. A table of values is given for a function $g(x)$. Determine if $g(x)$ is linear, quadratic, or neither.

x	-6	-4	-2	0	2	4
$g(x)$	10	11	15	24	40	65