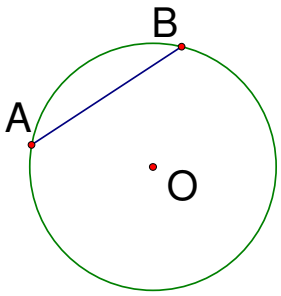
Exploring Chords and Arcs

with Geometer's Sketchpad

In this activity, we will investigate properties of chords and arcs within a circle.

Def. Chord – *a line segment whose endpoints are* _____



$\overline{AB}$ is a chord of circle *O.*

Construction note: *When making the following constructions, do not use the initial construction point on the circle as one of your endpoints for chords or arcs. Save this point as a handle to modify the size of the circle.*

Part 1: Central Angles, Arcs, and Chords

1. Construct circle *O* (label the center *O*).
2. Construct central angles $\angle AOB$ and $\angle COD$ with endpoints on the circle.
3. Measure the angles $\angle AOB$ and $\angle COD$. Move the points until the two angles are congruent.
4. Construct chords $\overline{AB}$ and $\overline{CD}$.
5. Measure these chords and complete this theorem with your observation:

   Theorem: Congruent central angles have _____ chords.
   Move the points *A, B, C,* and *D* to verify this.
6. Measure the lengths of arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ by selecting the circle and two endpoints, then go to the [Measure] menu.

   Theorem: Congruent chords have _____ arcs
   Move the points *A, B, C,* and *D* to verify this.
7. Finally use the transitive property to connect these two statements.

   Theorem: Congruent central angles have _____ arcs.
   Move the points *A, B, C,* and *D* to verify this.

Part 2: Equidistant Chords

1. Start a new construction and construct circle *P.*
2. Construct chords $\overline{AB}$ and $\overline{CD}$.
3. Construct a line Perpendicular to $\overline{AB}$ that passes through *P.*
4. Construct the intersection of $\overline{AB}$ and the perpendicular line and label it point *M.*
5. Construct a line Perpendicular to $\overline{CD}$ that passes through *P.*
6. Construct the intersection of $\overline{CD}$ and the perpendicular line and label it point *N.*
7. Measure the distances *MP* and *NP,* and the length of chords $\overline{AB}$ and $\overline{CD}$.
8. Move the points *A, B, C,* and *D* until the chords are congruent. Now complete the following theorem by observing your measurements

   Theorem: Congruent chords are _____ from the center.
   Move the points *A, B, C,* and *D* to verify this.

   Theorem: We can also say that Chords that are _____ from the center are congruent.

Part 3: Diameters and Chords

1. Start a new construction and construct circle Q.
2. Construct chord $\overline{AB}$
3. Construct the perpendicular bisector of chord $\overline{AB}$. (Hint: you will need to make midpoints first)
4. Construct the intersection points where this perpendicular bisector intersects the circle. Label these points C and D.

   What is special about segment $\overline{CD}$ (what kind of segment is it)?
   Move the points A *to* B to verify that this is always true.

   Theorem: In a circle the perpendicular bisector goes through the _____ of the circle.

Part 4: Diameters and Arcs

1. Continue with the drawing from part 3. Delete the perpendicular bisector and the midpoint of $\overline{AB}$
2. Construct point E on the circle.
3. Construct Line $\overleftrightarrow{EQ}$
4. Construct the intersection of this line and the circle (on the opposite side of E). Label this point F.

   What kind of segment is $\overline{EF}$?

5. Move E until $\overline{EF}$ intersects $\overline{AB}$. Construct this intersection point and label it G.
6. Measure $\angle AGQ$. Measure segments $\overline{AG}$ and $\overline{GB}$. Measure arcs $\widehat{AE}$, $\widehat{BE}$, $\widehat{AF}$, and $\widehat{BF}$.
7. Move E until the diameter is perpendicular to the chord. Complete the theorem:

   Theorem: If a diameter is perpendicular to a chord, then it _____ the chord and the arcs.
   Move the endpoints of the diameter and the chord to verify this.

Part 5 (Challenge): Find the center

1. Copy a picture of a circular object (such as the earth or a wheel) into Geometer's Sketchpad.
2. Construct two chords on this circle.
3. Use what you discovered in the previous parts to find the center of the circle using constructions. (Do not guess at the center by trying to draw a random circle that fits).
4. Once you have found and constructed the center. Verify that it is the center by construction a circle from the center to the outside of the circle in the picture. Does the circle perfectly fit the picture?