Calculus Name: Date:

2G Exercises

Derivatives of Inverse Trig Functions

Find the derivatives of these functions

43. $f(x) = 2\arcsin(x-1)$

$f'(x) = \dfrac{2}{\sqrt{1-(x-1)^2}}$

44. $f(t) = \arcsin t^2$

$f'(t) = \dfrac{2t}{\sqrt{1-(t^2)^2}} = \boxed{\dfrac{2t}{\sqrt{1-t^4}}}$

45. $g(x) = 3\arccos\dfrac{x}{2}$

$g'(x) = \dfrac{-3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} = \boxed{\dfrac{-3}{\sqrt{1-\frac{x^2}{4}}}}$

47. $f(x) = \arctan e^x$

$f'(x) = \dfrac{e^x}{1+(e^x)^2} = \boxed{\dfrac{e^x}{1+e^{2x}}}$

49. $g(x) = \dfrac{\arcsin 3x}{x}$

$g'(x) = \dfrac{\frac{3}{\sqrt{1-(3x)^2}}\cdot x - \arcsin 3x \cdot 1}{x^2}$ $\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$g'(x) = \dfrac{3}{x\sqrt{1-9x^2}} - \dfrac{\arcsin 3x}{x^2}$

50. $h(x) = x^2 \arctan 5x$

$h'(x) = 2x\arctan 5x + x^2\dfrac{5}{1+25x^2}$

$= \boxed{2x\arctan 5x + \dfrac{5x^2}{1+25x^2}}$

51. $h(t) = \sin(\arccos t)$

$h'(t) = \cos(\arccos t)\cdot\dfrac{-1}{\sqrt{1-t^2}}$

$= t\,\dfrac{-1}{\sqrt{1-t^2}}$

$= \boxed{\dfrac{-t}{\sqrt{1-t^2}}}$

57. $y = x\arcsin x + \sqrt{1-x^2}$

$y' = 1\arcsin x + x\cdot\dfrac{1}{\sqrt{1-x^2}} + \frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}(-\cancel{2}x)$

$= \arcsin x + \dfrac{x}{\sqrt{1-x^2}} - \dfrac{x}{\sqrt{1-x^2}}$

$= \boxed{\arcsin x}$ Cool!

56. $y = \dfrac{1}{2}\left[x\sqrt{4-x^2} + 4\arcsin\left(\dfrac{x}{2}\right)\right]$

$y' = \dfrac{1}{2}\left[1\sqrt{4-x^2} + x\,\frac{1}{\cancel{2}}(4-x^2)^{-\frac{1}{2}}\cdot(-\cancel{2}x) + 4\cdot\dfrac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}\right]$

$= \dfrac{1}{2}\left[\sqrt{4-x^2} - \dfrac{x^2}{\sqrt{4-x^2}} + \dfrac{2}{\sqrt{\frac{1}{4}}\sqrt{4-x^2}}\right]$

$= \dfrac{1}{2}\left[\sqrt{4-x^2} - \dfrac{x^2}{\sqrt{4-x^2}} + \dfrac{4}{\sqrt{4-x^2}}\right] = \dfrac{1}{2}\left[\dfrac{4-x^2}{\sqrt{4-x^2}} + \dfrac{4-x^2}{\sqrt{4-x^2}}\right] = \dfrac{4-x^2}{\sqrt{4-x^2}}\;\dfrac{\sqrt{4-x^2}}{\sqrt{4-x^2}} = \boxed{\sqrt{4-x^2}}$

You didn't need to simplify quite this far. But you should be able to

Unit Circle Time! Find the equation of the tangent line to the graph of y at the indicated point.

a. $y = \sec^{-1} x, \quad x = 2, 0 \le y \le \frac{\pi}{2}$

$y' = \dfrac{1}{|x|\sqrt{x^2-1}}$

$y' = \dfrac{1}{2\sqrt{4-1}} = \dfrac{1}{2\sqrt{3}}$

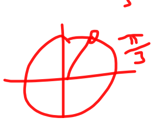
$\boxed{y - \frac{\pi}{3} = \frac{1}{2\sqrt{3}}(x-2)}$

$y = \sec^{-1}(2)$

$\sec y = 2$

$\cos y = \frac{1}{2}$

$y = \frac{\pi}{3}$



$\frac{\pi}{3}$

b. $y = \sin^{-1}\left(\frac{x}{2}\right), \quad x = \sqrt{3}, 0 \le y \le \frac{\pi}{2}$

$y' = \dfrac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}$

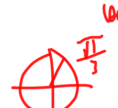
$x = \sqrt{3}$

$y' = \dfrac{\frac{1}{2}}{\sqrt{1-\frac{3}{4}}} = \dfrac{\frac{1}{2}}{\sqrt{\frac{1}{4}}} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = \boxed{1}$

$y = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$y = \frac{\pi}{3}$

$\boxed{\text{Point } \left(\sqrt{3}, \frac{\pi}{3}\right)}$



$\frac{\pi}{3}$

$\boxed{y - \frac{\pi}{3} = 1(x-\sqrt{3})}$ or $\boxed{y = x - \sqrt{3} + \frac{\pi}{3}}$

Larson section 5.6