Calculus

Name: Date:

2E Exercises

Implicit Differentiation Rules

Use Implicit differentiation to find the derivative of the function.

1. $x^2 + y^2 = 9$

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y}$$

3. $x^{1/2} + y^{1/2} = 16$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$y' = -\frac{x^{-1/2}}{y^{-1/2}}$$

$$= -\sqrt{\frac{y}{x}}$$

5. $x^3 - xy + y^2 = 7$

$$3x^2 - xy' - y + 2yy' = 0$$

$$(2y - x)y' = y - 3x^2$$

$$y' = \frac{y - 3x^2}{2y - x}$$

7. $x^3y^3 - y = x$

$$3x^3y^2y' + 3x^2y^3 - y' - 1 = 0$$

$$(3x^3y^2 - 1)y' = 1 - 3x^2y^3$$

$$y' = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

9. $x^3 - 3x^2y + 2xy^2 = 12$

$$3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 = 0$$

$$(4xy - 3x^2)y' = 6xy - 3x^2 - 2y^2$$

$$y' = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$$

11. $\sin x + 2\cos 2y = 1$

$$\cos x - 4(\sin 2y)y' = 0$$

$$y' = \frac{\cos x}{4 \sin 2y}$$

13. $\sin x = x(1 + \tan y)$

$$\cos x = x\left(\sec^2 y\right)y' + (1 + \tan y)(1)$$

$$y' = \frac{\cos x - \tan y - 1}{x \sec^2 y}$$

15. $y = \sin xy$

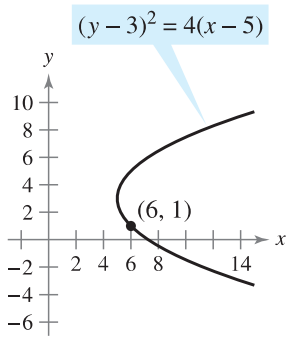
$$y' = [xy' + y]\cos(xy)$$

$$y' - x\cos(xy)y' = y\cos(xy)$$

$$y' = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

Find the equation of the tangent line at the given point. You can check this by graphing on Desmos

33. Parabola



$$2(y - 3)y' = 4$$

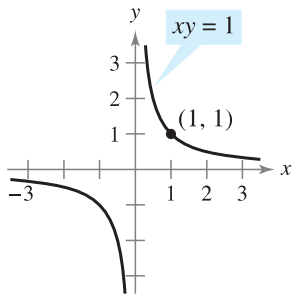
$$y' = \frac{2}{y - 3}$$

$$\text{At } (6, 1),\ y' = \frac{2}{1 - 3} = -1$$

$$\text{Tangent line: } y - 1 = -1(x - 6)$$

$$y = -x + 7$$

35. Rotated hyperbola



$$xy' + y = 0$$

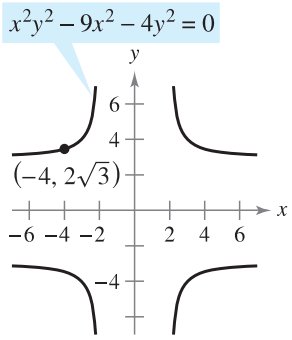
$$y' = \frac{-y}{x}$$

$$\text{At } (1, 1): y' = -1$$

$$\text{Tangent line: } y - 1 = -1(x - 1)$$

$$y = -x + 2$$

37. Cruciform



$$x^2 2yy' + 2xy^2 - 18x - 8yy' = 0$$

$$y' = \frac{18x - 2xy^2}{2x^2y - 8y}$$

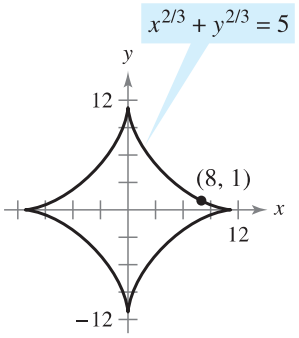
At $\left(-4, 2\sqrt{3}\right)$: $y' = \dfrac{18(-4) - 2(-4)(12)}{2(16)\left(2\sqrt{3}\right) - 16\sqrt{3}}$

$$= \frac{24}{48\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

Tangent line: $y - 2\sqrt{3} = \dfrac{\sqrt{3}}{6}(x + 4)$

$$y = \frac{\sqrt{3}}{6}x + \frac{8}{3}\sqrt{3}$$

38. Astroid



Find the second derivative in terms of y and x.

45. $x^2 + y^2 = 4$

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y}$$

$$y'' = \frac{y(-1) + xy'}{y^2} = \frac{-y + x(-x/y)}{y^2} = \frac{-y^2 - x^2}{y^3} = -\frac{4}{y^3}$$

47. $x^2 - y^2 = 36$

$$2x - 2yy' = 0$$

$$y' = \frac{x}{y}$$

$$x - yy' = 0$$

$$1 - yy'' - (y')^2 = 0$$

$$1 - yy'' - \left(\frac{x}{y}\right)^2 = 0$$

$$y^2 - y^3y'' = x^2$$

$$y'' = \frac{y^2 - x^2}{y^3} = -\frac{36}{y^3}$$