Calculus

Name: Date:

2B.2 Exercises

Derivative as Rate of Change

The following are problems from AP tests that refer to the derivative as the rate of change, or involve position-velocity-acceleration topics.

1. [1998 AP-MC#16 *no calculuator*]
   A particle moves along the $x-axis$ so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

2. [2017 AP-FR#5 *no calc*]
   No calculator – (except to find $x_p'(t)$ for part (a) because we don't know how to do that yet).
   Skip part (d)... that's 2nd semester stuff.
   Two particles move along the x-axis. For $0 \leq t \leq 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$.
   Particle Q is at position $x = 5$ at time $t = 0$.

   (a) For $0 \leq t \leq 8$, when is particle P moving to the left?

   (b) For $0 \leq t \leq 8$, find all times t during which the two particles travel in the same direction.

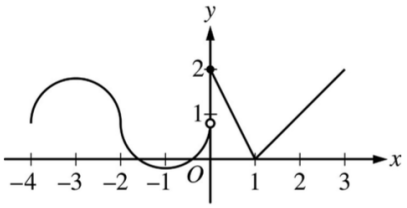
   (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

   Extra Challenge: you DON'T need to worry about doing this now, we need another tool. However, if you're clever you can figure this out with what you know. You need to find position functoion $x_Q(t)$.

   (d) Find the position of particle Q the first time it changes direction.

Not rate of change, but a couple good AP derivative questions

3. [2014-AP-sample#3]



Graph of f

The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x=-2$ and horizontal tangent lines at $x=-3$ and $x=-1$. What are all values of x, $-4<x<3$, at which f is continuous but not differentiable?

(A) $x=1$

(B) $x=-2$ and $x=0$

(C) $x=-2$ and $x=1$

(D) $x=0$ and $x=1$

4. [2014-AP-Sample#6]

$$f(x)=\begin{cases}2x-2 & \text{for } x<3\\ 2x-4 & \text{for } x\geq 3\end{cases}$$

Let f be the piecewise-linear function defined above. Which of the following statements are true?

I. $\lim\limits_{h\to 0^-}\dfrac{f(3+h)-f(3)}{h}=2$

II. $\lim\limits_{h\to 0^+}\dfrac{f(3+h)-f(3)}{h}=2$

III. $f'(3)=2$

(A) None

(B) II only

(C) I and II only

(D) I, II, and III

5. [2014-AP-Sample-#13]

The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5)=2$?

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.