2B.1 Exercises

Basic Differentiation Rules

Use the rules of differentiation to find the derivative of the function.

5. $y = x^7$

$$y' = 7x^6$$

7. $y = \frac{1}{x^5}$

$$y' = -5x^{-6} = -\frac{5}{x^6}$$

9. $f(x) = \sqrt[5]{x}$

$$y' = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$$

12. $g(x) = 3x - 1$

41. $g(t) = t^2 - \frac{4}{t^3}$

$$g'(t) = 2t + 12t^{-4} = 2t + \frac{12}{t^4}$$

45. $f(x) = \frac{x^3 - 3x^2 + 4}{x^2}$

$$f'(x) = 1 - \frac{8}{x^3} = \frac{x^3 - 8}{x^3}$$

49. $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

50. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$

$$f'(x) = \frac{1}{2}x^{-1/2} - 2x^{-2/3} = \frac{1}{2\sqrt{x}} - \frac{2}{x^{2/3}}$$

51. $h(s) = s^{4/5} - s^{2/3}$

$$h'(s) = \frac{4}{5}s^{-1/5} - \frac{2}{3}s^{-1/3} = \frac{4}{5s^{1/5}} - \frac{2}{3s^{1/3}}$$

In Exercises 55–58,
(a) find an equation of the tangent line to the graph of *f* at the given point,
(b) use a graphing utility to graph the function and its tangent line at the point, and
(c) use the *derivative* feature of a graphing utility to confirm your results.

55. $y = x^4 - 3x^2 + 2$ Point: (1,0)

(a) $y = x^4 - 3x^2 + 2$

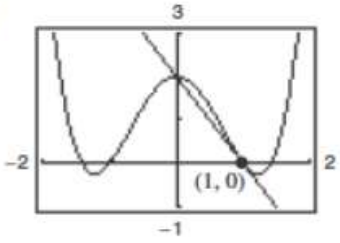
$y' = 4x^3 - 6x$

At $(1, 0)$: $y' = 4(1)^3 - 6(1) = -2$

Tangent line: $y - 0 = -2(x - 1)$

$2x + y - 2 = 0$

(b)



56. $y = x^3 + x$ Point (-1,-2)

57. $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$ Point (1,2)

(a) $f(x) = \dfrac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = -\dfrac{3}{2}x^{-7/4} = -\dfrac{3}{2x^{7/4}}$

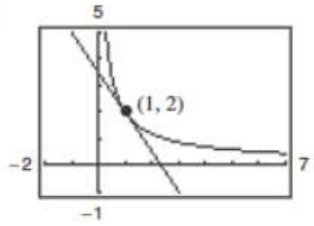
At $(1, 2)$: $f'(1) = -\dfrac{3}{2}$

Tangent line: $y - 2 = -\dfrac{3}{2}(x - 1)$

$y = -\dfrac{3}{2}x + \dfrac{7}{2}$

$3x + 2y - 7 = 0$

(b)



Use the rules of differentiation to find the derivative of the function.

19. $y = \dfrac{\pi}{2}\sin\theta - \cos\theta$

$y' = \dfrac{\pi}{2}\cos\theta + \sin\theta$

21. $y = x^2 - \frac{1}{2}\cos x$

$y' = 2x + \frac{1}{2}\sin x$

Find the slope of the graph of the function at the given point.
Use the *derivative* feature of a graphing utility to confirm your results.

37. $f(\theta) = 4 \sin \theta - \theta$ Point (0,0)

38. $g(t) = -2 \cos t + 5$ Point $(\pi, 7)$

$f'(\theta) = 4 \cos \theta - 1$

$f'(0) = 4(1) - 1 = 3$

Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

59. $y = x^4 - 2x^2 + 3$

$$\begin{aligned} y' &= 4x^3 - 4x \\ &= 4x(x^2 - 1) \\ &= 4x(x - 1)(x + 1) \end{aligned}$$

$y' = 0 \Rightarrow x = 0, \pm 1$

Horizontal tangents: $(0, 3), (1, 2), (-1, 2)$