Name: Solutions Date:

2A.1 Exercises

The Derivative and the Tangent Line Problem

Find $f'(x)$ using the formal definition of a derivative. Show all work.

1. $f(x) = 10x + 7$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\left(\frac{1}{h}\left(\,(10(x+h)+7)-(10x+7)\right)\right)$$
$$= \lim_{h\to 0}\left(\frac{1}{h}(\,10x+10h+7-10x-7)\right)$$
$$= \lim_{h\to 0}\left(\frac{1}{h}(\,10h)\right)$$
$$= \mathbf{10}$$

2. $f(x) = 6x - x^2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\left(\frac{1}{h}\left((6(x+h)-(x+h)^2)-(6x-x^2)\right)\right)$$

To be checked on assignment

3. $f(x) = \sqrt{3x+4}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\left(\frac{\left(\sqrt{3(x+h)+4}-\sqrt{3x+4}\right)}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{(3(x+h)+4)-(3x+4)}{h\left(\sqrt{3(x+h)+4}+\sqrt{3x+4}\right)}\right)$$
$$= \lim_{h\to 0}\left(\frac{3h}{h\left(\sqrt{3(x+h)+4}+\sqrt{3x+4}\right)}\right)$$
$$= \lim_{h\to 0}\left(\frac{3}{\left(\sqrt{3(x+h)+4}+\sqrt{3x+4}\right)}\right)$$
$$= \frac{3}{\left(\sqrt{3x+4}+\sqrt{3x+4}\right)} = \mathbf{\frac{3}{2\sqrt{3x+4}}}$$

4. $f(x) = \frac{2}{x+3}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\left(\frac{\frac{2}{(x+h)+3}-\frac{2}{x+3}}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{2(x+3)-2(x+h+3)}{h(x+h+3)(x+3)}\right)$$
$$= \lim_{h\to 0}\left(\frac{-2h}{h(x+h+3)(x+3)}\right)$$
$$= \lim_{h\to 0}\left(\frac{-2}{(x+h+3)(x+3)}\right)$$
$$= \frac{-2}{(x+3)(x+3)} = \mathbf{\frac{-2}{(x+3)^2}}$$

Find these derivatives directly

5. Find $f'(3)$ if $f(x) = 3x^2 - 5$

$$f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$
$$= \lim_{h\to 0}\frac{(3(3+h)^2-5)-(3(3^2)-5)}{h}$$
$$= \lim_{h\to 0}\frac{(3(9+6h+h^2)-5)-22}{h}$$
$$= \lim_{h\to 0}\frac{18h+3h^2}{h} = \lim_{h\to 0} 18+3h = \mathbf{18}$$

6. Find $f'(2)$ if $f(x) = \sqrt{x+5}$

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$= \lim_{h\to 0}\frac{\sqrt{2+h+5}-\sqrt{2+5}}{h}$$
$$= \lim_{h\to 0}\frac{7+h-7}{h\left(\sqrt{2+h+5}+\sqrt{2+5}\right)}$$
$$= \lim_{h\to 0}\frac{1}{\left(\sqrt{7+h}+\sqrt{7}\right)} = \frac{1}{2\sqrt{7}}$$