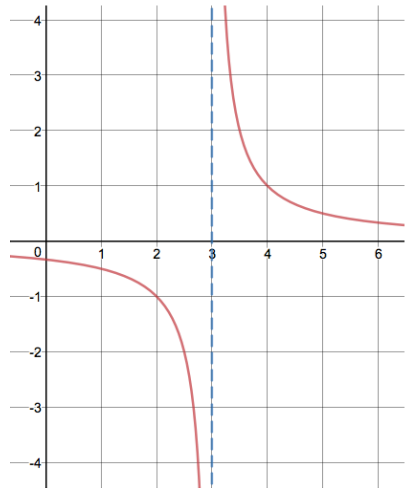
Name: Date:

1D: Infinite Limits and Limits at Infinity

Now that we have a handle on limits of functions that are approaching finite values, we will turn our attention to our last topic (for now) regarding limits… *Infinite limits.*



First, we need to revisit the case of an asymptote of a rational function. Consider this limit $\lim_{x\to 3} \frac{1}{x-3}$. By the definition of a limit, this limit Does Not Exist (DNE). However, we will describe the one-sided limits differently…

$$\lim_{x\to 3^-} \frac{1}{x-3} = \qquad\qquad \lim_{x\to 3^+} \frac{1}{x-3} =$$

Watch out! If a limit is equal to ∞ or $-\infty$, it doesn't mean the limit exists, it just tells us a little more about the trend of the function.

Limits at Infinity

What if the value of x is moving toward $\pm\infty$? We can find out if these limits exist or not. Let's try it!

<u>Example</u> 1 Find these limits.

$$\lim_{x\to\infty} \frac{4}{x-5} = \qquad\qquad \lim_{x\to -\infty} \frac{2000000}{x^3-2} =$$

Key Point: When a value of x becomes a very large positive or negative number, other smaller numbers become "worthless".

> ***Consider this:*** Suppose you win a trillion dollars in the lottery! That's $1,000,000,000,000! Would you care if you lost $1? How about $100? How about $1000, or even $1,000,000? If you lost 1 million dollars, it shouldn't matter because you have "a million millions"!

So, when we are finding limits of rational functions as x$\to 0$, the small terms don't matter… so we can eliminate them to get a simpler limit that we can easily evaluate.

<u>Example 2:</u> Find the limit.

a. $\lim_{x\to -\infty} \frac{3x-4}{7-5x} =$

b. $\lim_{x\to\infty} \frac{4-x}{3x-5} =$

Strategy for Infinite Limits of Rational Functions:
eliminate lower powers in numerator and denominator, then evaluate.

c. $\lim\limits_{x\to\infty} \frac{4-3x^2+2x}{4x+2} =$

d. $\lim\limits_{x\to-\infty} \frac{4-3x^2+2x}{4x+2} =$

Okay, here are some challenging ones to think about.

e. $\lim\limits_{x\to-\infty} e^{-x} =$

f. $\lim\limits_{x\to\infty} \frac{-2\sqrt{9x^{10}+2x^5+5}}{-12x^5+3x^3-2x^2-1} =$

Here's a useful theorem that says if $f(x)$ is between two functions that have the same limit, this must be the limit of $f(x)$. More formally:

Squeeze Theorem

If $p(x) \le f(x) \le q(x)$ and $\lim\limits_{x\to c} p(x) = L = \lim\limits_{x\to c} q(x)$, then $\lim\limits_{x\to c} f(x) = L$.

Example: Find this limit:

$$\lim_{x\to\infty} \frac{1}{x}\sin x$$

One more use of the squeeze theorem..

For $\theta \to 0$, using the unit circle we have

$$\sin\theta \le \theta \le \tan\theta$$