1B Exercises

Finding Limits Analytically

Complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result. Sketch the graph in the neighborhood of the limit point.

Find the limit.

19. $\lim\limits_{x \to 1} \dfrac{x}{x^2 + 4}$

21. $\lim\limits_{x \to 7} \dfrac{3x}{\sqrt{x + 2}}$

Use the information to evaluate the limits

39. $\lim\limits_{x \to c} f(x) = 4$

(a) $\lim\limits_{x \to c} [f(x)]^3$

(b) $\lim\limits_{x \to c} \sqrt{f(x)}$

(c) $\lim\limits_{x \to c} [3f(x)]$

(d) $\lim\limits_{x \to c} [f(x)]^{3/2}$

Find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

45. $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

46. $\lim\limits_{x \to -1} \dfrac{2x^2 - x - 3}{x + 1}$

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47. $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

49. $\lim\limits_{x \to 0} \dfrac{x}{x^2 - x}$

52. $\lim\limits_{x \to 3} \dfrac{3 - x}{x^2 - 9}$

53. $\lim\limits_{x \to -3} \dfrac{x^2 + x - 6}{x^2 - 9}$

55. $\lim\limits_{x \to 4} \dfrac{\sqrt{x + 5} - 3}{x - 4}$

58. $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$

59. $\lim\limits_{x \to 0} \dfrac{[1/(3 + x)] - (1/3)}{x}$

60. $\lim\limits_{x \to 0} \dfrac{[1/(x + 4)] - (1/4)}{x}$

Determine the limit of the trigonometric function (if it exists).

65. $\lim\limits_{x \to 0} \dfrac{\sin x}{5x}$

66. $\lim\limits_{x \to 0} \dfrac{3(1 - \cos x)}{x}$

71. $\lim\limits_{h \to 0} \dfrac{(1 - \cos h)^2}{h}$

68. $\lim\limits_{\theta \to 0} \dfrac{\cos \theta \tan \theta}{\theta}$

73. $\displaystyle\lim_{x\to\pi/2} \frac{\cos x}{\cot x}$

74. $\displaystyle\lim_{x\to\pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

76. $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$ $\left[Hint: \text{Find } \displaystyle\lim_{x\to 0}\left(\frac{2 \sin 2x}{2x}\right)\left(\frac{3x}{3 \sin 3x}\right). \right]$

Optional Challenge Problem.
Hint for part (d): Multiply the bottom by $\dfrac{\sec x+1}{\sec x+1}$ *and then remember that* $\sec^2 x - 1 = \tan^2 x$

125. *Graphical Reasoning*   Consider $f(x) = \dfrac{\sec x - 1}{x^2}$.

    (a) Find the domain of f.

    (b) Use a graphing utility to graph f. Is the domain of f obvious from the graph? If not, explain.

    (c) Use the graph of f to approximate $\displaystyle\lim_{x\to 0} f(x)$.

    (d) Confirm your answer to part (c) analytically.