Name: Date:

1A: Finding Limits Graphically and Numerically

Exploration

Suppose you have an empty gallon jug and you fill it with $\frac{1}{2}$ gallon of water the first day, then $\frac{1}{4}$ gallon the second day, then $\frac{1}{8}$ gallon, then $\frac{1}{16}$ gallon, and so on. If you continue this pattern, when will he need to get a 2nd milk jug to hold?

When there is a number that a function (or a series like the one above) gets very close to, but may never actually touch, this is called a limit.

Using Tables and Graphs

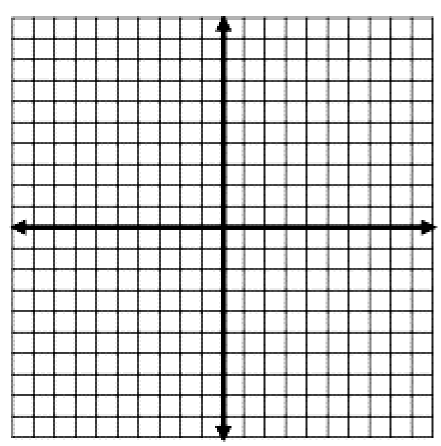
Let's consider a limit of a rational function. Take the function $f(x) = \frac{x-2}{x^2+x-6}$,

a) Find $f(2)$.

b) Let's find out what the value of f is as we get real close to 2. Compute these from the outside-in. (Note, this is a good time to use the [$sto\ \rightarrow$] button on your calculator to define a function.)

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$							

What is the value that $f(x)$ approaches as x approaches 2?
We call this the "limit as x approaches 2" and write this $\lim\limits_{x\to 2} f(x)$.



c) Graph the equation by hand, then check with a calculator. It may help to rewrite the function as

$$f(x) = \frac{x-2}{(\mathrm{x}+3)(\mathrm{x}-2)}$$

d) Use your calculator to graph the function. How does this help us find the limit as $x \rightarrow 2$?

What is a limit?

A limit is the value of a function as it gets arbitrarily close a given value of x. It's what $f(x)$ is *approaching* as x gets close to c, even if the actual value of $f(c)$ is not equal to the limit.

> Definition:
> If $f(x)$ becomes arbitrarily close (i.e. really close) to a single number L as x approaches c from either side, then the limit of $\boldsymbol{f(x)}$, as x approaches c, is L. This is written as
>
> $$\lim f(x) = L$$

Three Types of limits	• **Left hand limit:** As x approaches c from the left side. Written $\lim\limits_{x \to c^-} f x)$ • **Right hand limit**: As x approaches c from the right side. Written $\lim\limits_{x \to c^+} f(x)$ • **Function limit at $\boldsymbol{x = c}$:** requires two things: 1. $\lim\limits_{x \to c^-} f x) = L$ 2. $\lim\limits_{x \to c^+} f x) = L$

In other words… the limit exists if the function is approaching the same value L from the left and right.

Try it Use a graphing calculator to evaluate these limits fOR $f(x) = \frac{x-1}{\sqrt{x}-1}$ and $g(x) = \frac{x-1}{x^2-1}$

a) $\lim\limits_{x \to 1^-} f(x) =$ b) $\lim\limits_{x \to 1^+} f(x) =$ c) $\lim\limits_{x \to 1} f(x) =$

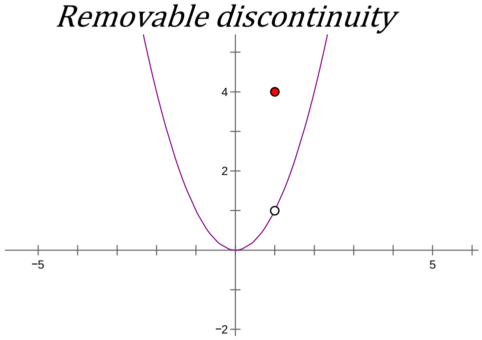
d) $\lim\limits_{x \to 1^-} g(x) =$ e) $\lim\limits_{x \to 1^+} g(x) =$ f) $\lim\limits_{x \to 1} g(x) =$

Limits and discontinuities

When we have discontinuities (breaks in a function) we will often want to find the limit as we approach the point of discontinuity.

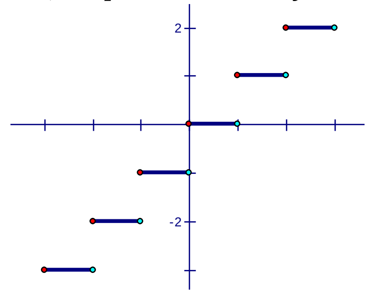
Types of discontinuity

Removable discontinuity



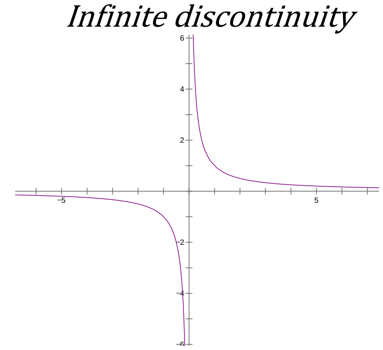
$\lim\limits_{x \to 1} f(x) = 1$

Jump discontinuity



$\lim\limits_{x \to 1} f(x)$ Does Not Exist because it's not the same from both sides

Infinite discontinuity



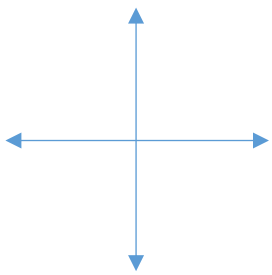
$\lim\limits_{x \to 0} f(x)$ Does Not Exist because it's not the same from both sides

Limits that fail to exist

Let's consider a few different cases of limits that cause trouble.
Graph the functions below and find the given limit if you can.

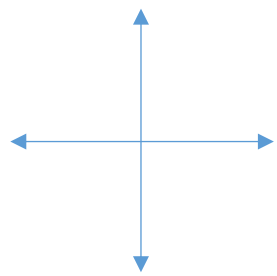
$$g(x) = \begin{cases} x+1, x \geq 0 \\ x-1, x < 0 \end{cases}$$

Find $\lim_{x \to 0} g(x)$



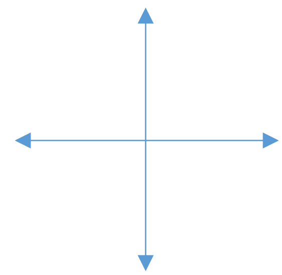
$$h(x) = \frac{1}{x^3}$$

Find $\lim_{x \to 0} h(x)$



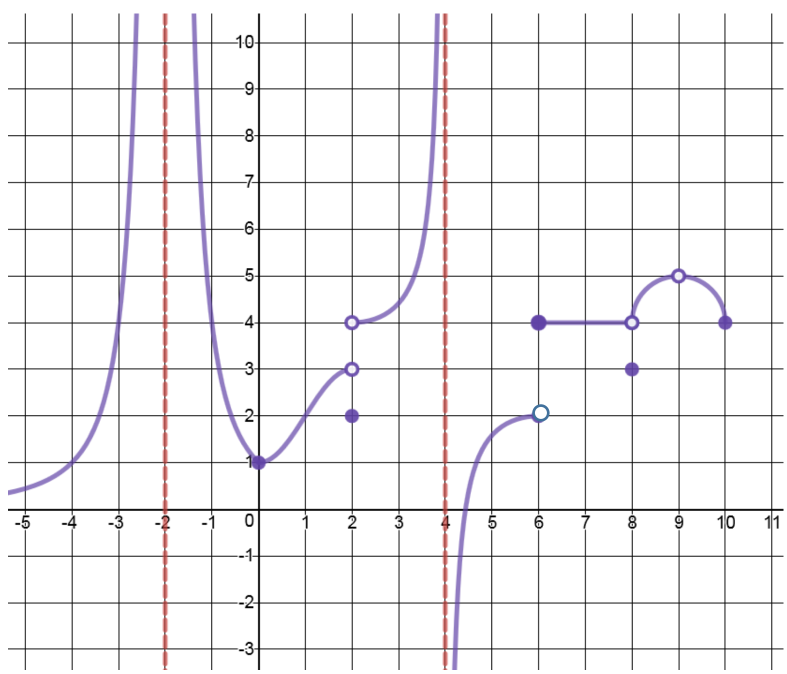
$$k(x) = \sin\frac{1}{x}$$

$\lim_{x \to 0} k(x)$



x	$2/\pi$	$2/3\pi$	$2/5\pi$	$2/7\pi$	$2/9\pi$	$2/11\pi$
$\sin(1/x)$	1	-1	1	-1	1	-1

It's important to understand that **there is a difference between the value of a function and the limit of a function**. Sometimes they are the same, and sometimes they are different. The value of a function is what happens at a point, the limit is what happens as it approaches that point. Use the graph of $f(x)$ below to evaluate the following.



a) $\lim_{x \to 0^-} f(x) =$

b) $\lim_{x \to 0^+} f(x) =$

c) $f(0) =$

d) $\lim_{x \to 2^-} f(x) =$

e) $\lim_{x \to 2^+} f(x) =$

f) $f(2) =$

g) $\lim_{x \to 4^-} f(x) =$

h) $\lim_{x \to 4^+} f(x) =$

i) $f(4) =$

j) $\lim_{x \to 6^-} f(x) =$

k) $\lim_{x \to 6^+} f(x) =$

l) $f(6) =$

m) $\lim_{x \to -2} f(x)$

Let's take this one step further…

Here is a very technical definition… this is complicated, and you won't be tested on it…. This year!

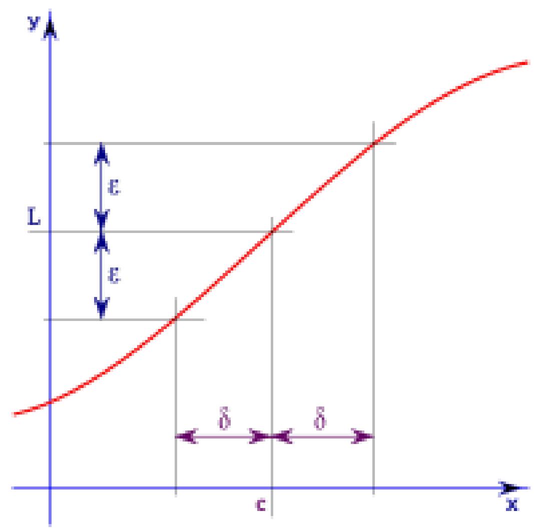
Epsilon-Delta $\boldsymbol{\varepsilon}$–$\boldsymbol{\delta}$ Definition of a Limit:

Let (x) be a function defined on an open interval including c (except possibly at the value c, and let L be a real number. Then we can say

$$\lim_{x \to c} f(x) = L$$

if for any small number $\varepsilon > 0$, there exists a $\delta > 0$ such that $0 < |x - c| < \delta$, *then*

$$f < |x - L| < \varepsilon$$



<u>**Example**</u> Back to our rational function

Consider the function $f(x) = \frac{x^3-2x^2}{x-2}$, use the $\boldsymbol{\varepsilon}$–$\boldsymbol{\delta}$ Definition to show that $\lim_{x \to 2} f(x) = 4$.

Find δ such that $\left|\frac{x^3-2x^2}{x-2} - 4\right| < .001$ when $0 < |x - 2| < \delta$.

In other words, find out how close does the x *value have to be to 2 to make the value of* $f(x)$ *within .001 of 4?*