Calculus Name: Date:

# 7A: Integration by Parts

**Consider These Integrals:** Can we use $u$-substitution or other techniques we've learned to find these integrals?

$$\int x^2 \ln x \; dx \;, \qquad \int x\, e^x \, dx \;, \qquad \int e^x \sin x \; dx$$

If you look closely, you'll notice that each of these are products of functions. So, let's remember our product rule:

$$\frac{d}{dx}[uv] = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx} = u'v + uv'$$

If we integrate both sides we obtain

$$uv = \int vu'dx + \int uv'dx = \int v\, du + \int u\, dv$$

Rewriting this we get this theorem.

**Integration by Parts**

If an integral is in the form $\int u\, dv$ for some differentiable functions $u(x)$ and $v(x)$, use

$$\int u\, dv = uv - \int v\, du$$

The key to using the Integration by Parts rule is to carefully define $u$ and $dv$

**GUIDELINES FOR INTEGRATION BY PARTS**

1. Try letting $dv$ be the most complicated portion of the integrand that fits a basic integration rule. Then $u$ will be the remaining factor(s) of the integrand.
2. Try letting $u$ be the portion of the integrand whose derivative is a function simpler than $u$. Then $dv$ will be the remaining factor(s) of the integrand.

Note that $dv$ always includes the $dx$ of the original integrand.

**Example 1** Integrate

$$\int xe^x$$

| Option 1 | | Option 2 | | Option 3 | | Option 4 | |
|---|---|---|---|---|---|---|---|
| $u =$ | $dv =$ | $u =$ | $dv =$ | $u =$ | $dv =$ | $u =$ | $dv =$ |
| $du=$ | $v=$ | $du=$ | $v=$ | $du=$ | $v=$ | $du=$ | $v=$ |

**Example 2:** Integrate

$$\int x^2 \ln x \ dx$$

**Example 3:** Integrate a single factor

$$\int \arcsin x \ dx$$

*Think of this as* $1 \cdot \arcsin x$.
*Since you don't know how to integrate* $\arcsin x$, *let* $u = \arcsin x$

**Example 4:** Repeat Integration by Parts.
*If at first you don't succeed, try, try it again!*

$$\int x^2 \sin x \; dx$$

**SUMMARY OF COMMON INTEGRALS USING INTEGRATION BY PARTS**

**1.** For integrals of the form

$$\int x^n e^{ax}\, dx, \qquad \int x^n \sin ax\, dx, \qquad \text{or} \qquad \int x^n \cos ax\, dx$$

let $u = x^n$ and let $dv = e^{ax}\, dx$, $\sin ax\, dx$, or $\cos ax\, dx$.

**2.** For integrals of the form

$$\int x^n \ln x\, dx, \qquad \int x^n \arcsin ax\, dx, \qquad \text{or} \qquad \int x^n \arctan ax\, dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\, dx$.

**3.** For integrals of the form

$$\int e^{ax} \sin bx\, dx \qquad \text{or} \qquad \int e^{ax} \cos bx\, dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\, dx$.