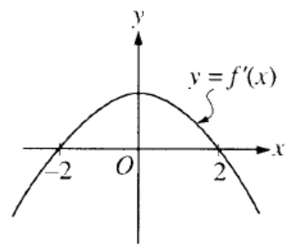
Name: Date:

AP Calculus A/B Practice Test #5

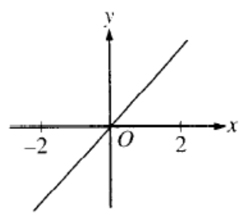
<u>Multiple Choice (Goal: 3 min. each)</u> ***1997***

No Calculator

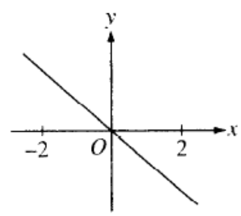


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

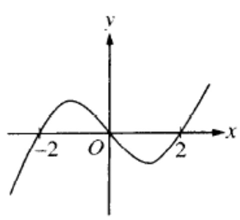
(A)



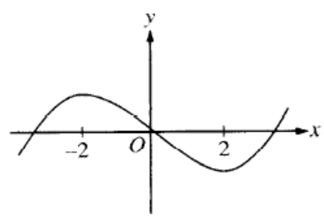
(B)



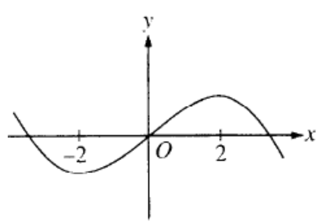
(C)



(D)



(E)



12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

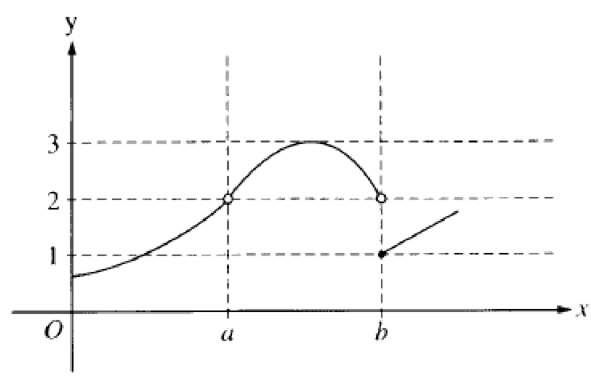
(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

13. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

19. If $f(x) = \ln\left| x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2-1}\right|$

(B) $\frac{2x}{\left|x^2-1\right|}$

(C) $\frac{2|x|}{x^2-1}$

(D) $\frac{2x}{x^2-1}$

(E) $\frac{1}{x^2-1}$

20. The average value of $\cos x$ on the interval $[-3, 5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

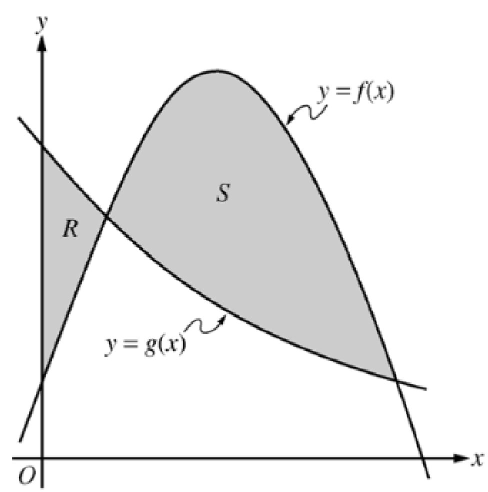
(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

Free Response (Goal: 15 min. each)

AP-2005-CR



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

AP-2005-CR

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

AP 2005-NC

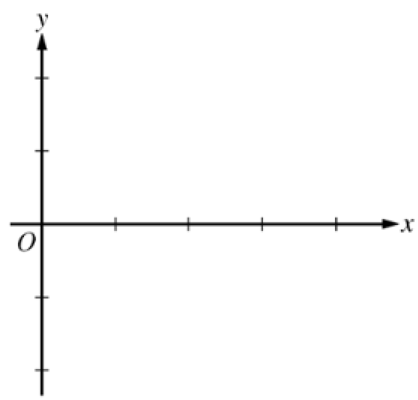
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

   (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

   (Note: Use the axes provided in the pink test booklet.)



   (c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

1997 Calculus AB Solutions: Part A

11. E Since f' is positive for $-2 < x < 2$ and negative for $x < -2$ and for $x > 2$, we are looking for a graph that is increasing for $-2 < x < 2$ and decreasing otherwise. Only option E.

12. B $y = \frac{1}{2}x^2$; $y' = x$; We want $y' = \frac{1}{2} \Rightarrow x = \frac{1}{2}$. So the point is $\left(\frac{1}{2}, \frac{1}{8}\right)$.

13. A $f'(x) = \frac{|4 - x^2|}{x - 2}$; f is decreasing when $f' < 0$. Since the numerator is non-negative, this is only when the denominator is negative. Only when $x < 2$.

14. C $f(x) \approx L(x) = 2 + 5(x - 3)$; $L(x) = 0$ if $0 = 5x - 13 \Rightarrow x = 2.6$

15. B Statement B is true because $\lim_{x \to a^-} f(x) = 2 = \lim_{x \to a^+} f(x)$. Also, $\lim_{x \to b} f(x)$ does not exist because the left- and right-sided limits are not equal, so neither (A), (C), nor (D) are true.

16. D The area of the region is given by $\int_{-2}^{2} (5 - (x^2 + 1))dx = 2(4x - \frac{1}{3}x^3)\Big|_0^2 = 2\left(8 - \frac{8}{3}\right) = \frac{32}{3}$

17. A $x^2 + y^2 = 25$; $2x + 2y \cdot y' = 0$; $x + y \cdot y' = 0$; $y'(4,3) = -\frac{4}{3}$;

$x + y \cdot y' = 0 \Rightarrow 1 + y \cdot y'' + y' \cdot y' = 0$; $1 + (3)y'' + \left(-\frac{4}{3}\right) \cdot \left(-\frac{4}{3}\right) = 0$; $y'' = -\frac{25}{27}$

18. C $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx$ is of the form $\int e^u du$ where $u = \tan x$..

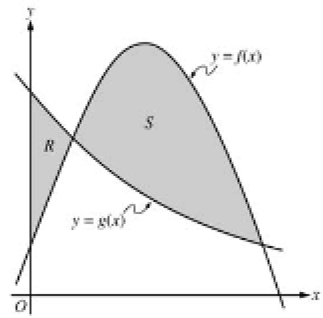
$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx = e^{\tan x}\Big|_0^{\frac{\pi}{4}} = e^1 - e^0 = e - 1$

19. D $f(x) = \ln|x^2 - 1|$; $f'(x) = \frac{1}{x^2 - 1} \cdot \frac{d}{dx}(x^2 - 1) = \frac{2x}{x^2 - 1}$

20. E $\frac{1}{8}\int_{-3}^{5} \cos x \, dx = \frac{1}{8}(\sin 5 - \sin(-3)) = \frac{1}{8}(\sin 5 + \sin 3)$; Note: Since the sine is an odd function, $\sin(-3) = -\sin(3)$.

Question 1

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

$f(x) = g(x)$ when $\frac{1}{4} + \sin(\pi x) = 4^{-x}$.

f and g intersect when $x = 0.178218$ and when $x = 1$.
Let $a = 0.178218$.

(a) $\int_0^a (g(x) - f(x))\,dx = 0.064$ or 0.065

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\int_a^1 (f(x) - g(x))\,dx = 0.410$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\pi\int_a^1 \left((f(x)+1)^2 - (g(x)+1)^2\right) dx = 4.558$ or 4.559

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

Question 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

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(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 R(t)\,dt = 31.815$ or $31.816\ \text{yd}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or $-1.909\ \text{yd}^3/\text{hr}$

$1 : \text{answer}$

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

$3 : \begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

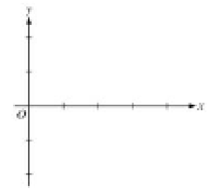
Question 4

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f. **(Note: Use the axes provided in the pink test booklet.)**



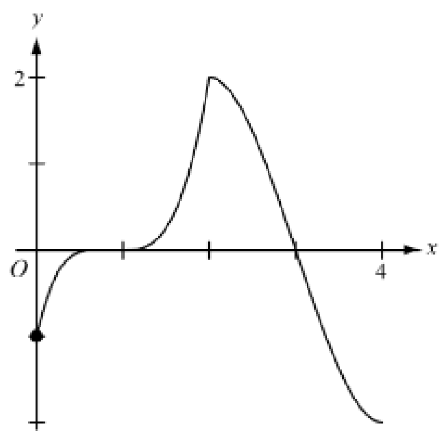
(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

(a) f has a relative maximum at $x = 2$ because f' changes from positive to negative at $x = 2$.

$2 : \begin{cases} 1 : \text{relative extremum at } x = 2 \\ 1 : \text{relative maximum with justification} \end{cases}$

(b)



$2 : \begin{cases} 1 : \text{points at } x = 0,\ 1,\ 2,\ 3 \text{ and behavior at } (2, 2) \\ 1 : \text{appropriate increasing/decreasing and concavity behavior} \end{cases}$

(c) $g'(x) = f(x) = 0$ at $x = 1, 3$.
g' changes from negative to positive at $x = 1$ so g has a relative minimum at $x = 1$. g' changes from positive to negative at $x = 3$ so g has a relative maximum at $x = 3$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{critical points} \\ 1 : \text{answer with justification} \end{cases}$

(d) The graph of g has a point of inflection at $x = 2$ because $g'' = f'$ changes sign at $x = 2$.

$2 : \begin{cases} 1 : x = 2 \\ 1 : \text{answer with justification} \end{cases}$