Name: Date:

AP Calculus A/B Practice Test #4

Multiple Choice (Goal: 3 min. each) 1997
Calculator Required

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

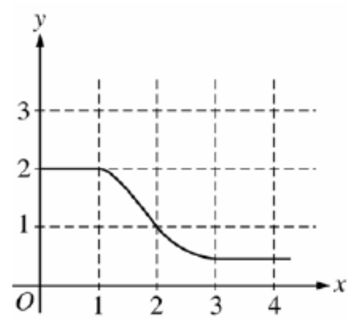
(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

80. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

82. If $y = 2x - 8$, what is the minimum value of the product xy?

(A) –16 (B) –8 (C) –4 (D) 0 (E) 2

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

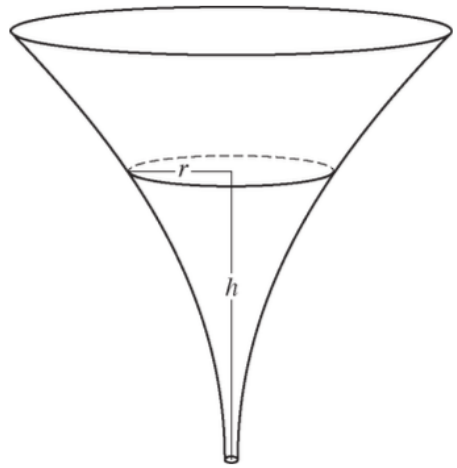
85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) –0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

Free Response (Goal: 15 min. each)

2016

NC



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

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76. E $f(x)=\dfrac{e^{2x}}{2x}$; $f'(x)=\dfrac{2e^{2x}\cdot 2x-2e^{2x}}{4x^2}=\dfrac{e^{2x}(2x-1)}{2x^2}$

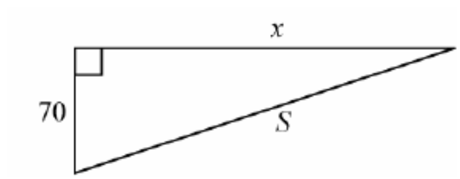
77. D $y=x^3+6x^2+7x-2\cos x$. Look at the graph of $y''=6x+12+2\cos x$ in the window $[-3,-1]$ since that domain contains all the option values. y'' changes sign at $x=-1.89$.

78. D $F(3)-F(0)=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=2+2.3=4.3$
(Count squares for $\int_0^1 f(x)dx$)

79. C The stem of the questions means $f'(2)=5$. Thus f is differentiable at $x=2$ and therefore continuous at $x=2$. We know nothing of the continuity of f'. I and II only.

80. A $f(x)=2e^{4x^2}$; $f'(x)=16xe^{4x^2}$; We want $16xe^{4x^2}=3$. Graph the derivative function and the function $y=3$, then find the intersection to get $x=0.168$.

81. A Let x be the distance of the train from the crossing. Then $\dfrac{dx}{dt}=60$.
$S^2=x^2+70^2 \Rightarrow 2S\dfrac{dS}{dt}=2x\dfrac{dx}{dt}\Rightarrow \dfrac{dS}{dt}=\dfrac{x}{S}\dfrac{dx}{dt}$.
After 4 seconds, $x=240$ and so $S=250$.
Therefore $\dfrac{dS}{dt}=\dfrac{240}{250}(60)=57.6$



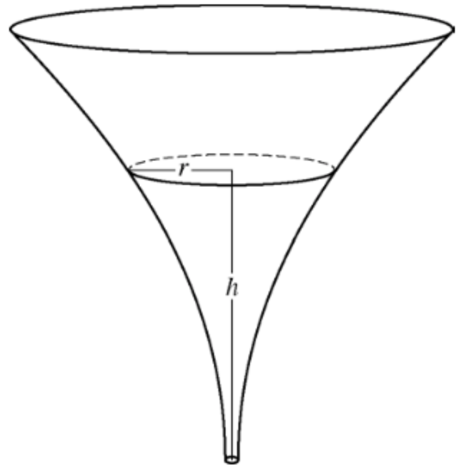
82. B $P(x)=2x^2-8x$; $P'(x)=4x-8$; P' changes from negative to positive at $x=2$. $P(2)=-8$

83. C $\cos x=x$ at $x=0.739085$. Store this in A. $\int_0^A(\cos x-x)\,dx=0.400$

84. C Cross sections are squares with sides of length y.
Volume $=\int_1^e y^2dx=\int_1^e \ln x\,dx=(x\ln x-x)\Big|_1^e=(e\ln e-e)-(0-1)=1$

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x=0.91$

Question 5



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(a) Average radius $= \frac{1}{10}\int_0^{10} \frac{1}{20}(3 + h^2)\,dh = \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$

$= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - 0\right) = \frac{109}{60}$ in

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^{10} \left(\left(\frac{1}{20}\right)(3 + h^2)\right)^2 dh = \frac{\pi}{400}\int_0^{10} (9 + 6h^2 + h^4)\,dh$

$= \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10}$

$= \frac{\pi}{400}\left(\left(90 + 2000 + \frac{100000}{5}\right) - 0\right) = \frac{2209\pi}{40}$ in^3

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec

$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
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(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

(a) $k(3) = f(g(3)) = f(6) = 4$

$k'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10$

An equation for the tangent line is $y = 10(x - 3) + 4$.

$3 : \begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$

(b) $h'(1) = \dfrac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2}$

$= \dfrac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \dfrac{-54}{36} = -\dfrac{3}{2}$

$3 : \begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$

(c) $\int_1^3 f''(2x)\,dx = \frac{1}{2}\left[f'(2x)\right]_1^3 = \frac{1}{2}[f'(6) - f'(2)]$

$= \frac{1}{2}[5 - (-2)] = \frac{7}{2}$

$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$