Calculus Name: Date:

AP Calculus A/B Practice Test #3

Multiple Choice (Goal: 3 min. each) *1997*

No Calculator

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{2\sqrt{2x-3}}$

3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

4. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) –1 (D) –3 (E) –5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

6. $\frac{1}{2}\int e^{\frac{t}{2}}\, dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

7. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

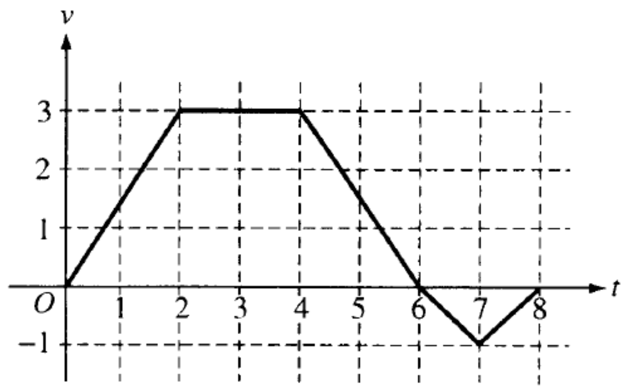
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \dfrac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \dfrac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \dfrac{\pi}{4}\right)$

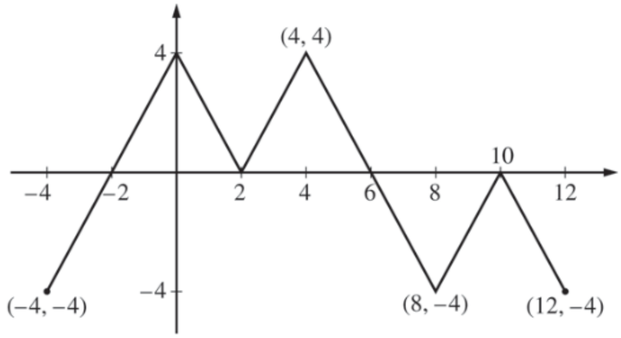
(C) $y = 2\left(x - \dfrac{\pi}{4}\right)$

(D) $y = -\left(x - \dfrac{\pi}{4}\right)$

(E) $y = -2\left(x - \dfrac{\pi}{4}\right)$

Free Response (Goal: 15 min. each)

NC
2016



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

   (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

   (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

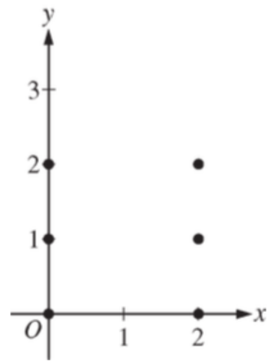
   (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

   (d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

NC
2016

4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

1997 Calculus AB Solutions: Part A

1. C $\int_1^2 (4x^3 - 6x)\,dx = (x^4 - 3x^2)\Big|_1^2 = (16-12)-(1-3) = 6$

2. A $f(x) = x(2x-3)^{\frac{1}{2}}$; $f'(x) = (2x-3)^{\frac{1}{2}} + x(2x-3)^{-\frac{1}{2}} = (2x-3)^{-\frac{1}{2}}(3x-3) = \dfrac{(3x-3)}{\sqrt{2x-3}}$

3. C $\int_a^b (f(x)+5)\,dx = \int_a^b f(x)dx + 5\int_a^b 1\,dx = a + 2b + 5(b-a) = 7b - 4a$

4. D $f(x) = -x^3 + x + \dfrac{1}{x}$; $f'(x) = -3x^2 + 1 - \dfrac{1}{x^2}$; $f'(-1) = -3(-1)^2 + 1 - \dfrac{1}{(-1)^2} = -3+1-1 = -3$

5. E $y = 3x^4 - 16x^3 + 24x^2 + 48$; $y' = 12x^3 - 48x^2 + 48x$; $y'' = 36x^2 - 96x + 48 = 12(3x-2)(x-2)$
   $y'' < 0$ for $\dfrac{2}{3} < x < 2$, therefore the graph is concave down for $\dfrac{2}{3} < x < 2$

6. C $\dfrac{1}{2}\int e^{\frac{t}{2}}\,dt = e^{\frac{t}{2}} + C$

7. D $\dfrac{d}{dx}\cos^2(x^3) = 2\cos(x^3)\left(\dfrac{d}{dx}(\cos(x^3)\right) = 2\cos(x^3)(-\sin(x^3)\left(\dfrac{d}{dx}(x^3)\right)$
   $= 2\cos(x^3)(-\sin(x^3)(3x^2)$

8. C The bug change direction when v changes sign. This happens at $t = 6$.

9. B Let A_1 be the area between the graph and t-axis for $0 \le t \le 6$, and let A_2 be the area between the graph and the t-axis for $6 \le t \le 8$ Then $A_1 = 12$ and $A_2 = 1$. The total distance is $A_1 + A_2 = 13$.

10. E $y = \cos(2x)$; $y' = -2\sin(2x)$; $y'\left(\dfrac{\pi}{4}\right) = -2$ and $y\left(\dfrac{\pi}{4}\right) = 0$; $y = -2\left(x - \dfrac{\pi}{4}\right)$

Question 3

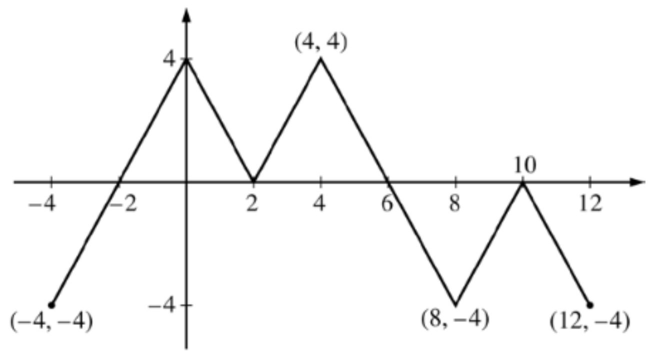
The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.



Graph of f

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1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

(a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \le 0$ for $8 \le x \le 12$.

1 : answer with justification

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

1 : answer with justification

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
−4	−4
−2	−8
6	8
12	−4

On the interval $-4 \le x \le 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

4 : { 1 : considers $x = -2$ and $x = 6$ as candidates; 1 : considers $x = -4$ and $x = 12$; 2 : answers with justification }

(d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$.

2 : intervals

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Question 4

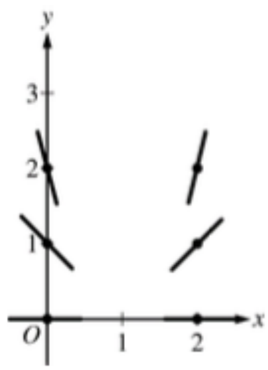
Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = \frac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x-2) + 3$.

$f(2.1) \approx 9(2.1-2) + 3 = 3.9$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{1}{y^2}\,dy = \frac{1}{x-1}\,dx$

$\int \frac{1}{y^2}\,dy = \int \frac{1}{x-1}\,dx$

$-\frac{1}{y} = \ln|x-1| + C$

$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$

$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$

$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables