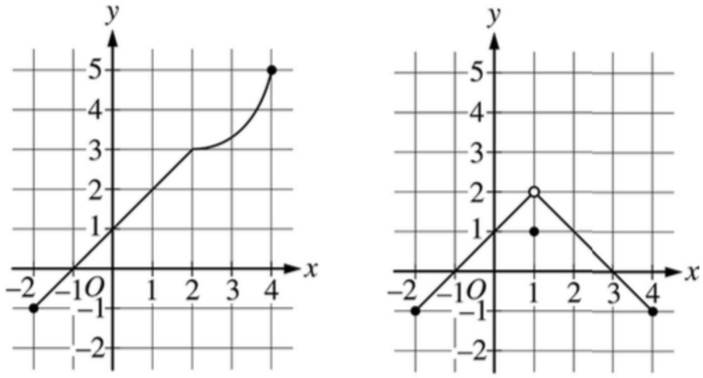
Name: Date:

AP Calculus A/B Practice Test #1

Multiple Choice (Goal: 2 min. each)

NC



Graph of f Graph of g

1. The graphs of the functions f and g are shown above. The value of $\lim_{x \to 1} f(g(x))$ is

   (A) 1

   (B) 2

   (C) 3

   (D) nonexistent

NC

2. $\lim_{x \to 0} \dfrac{7x - \sin x}{x^2 + \sin(3x)} =$

   (A) 6

   (B) 2

   (C) 1

   (D) 0

NC

3. If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

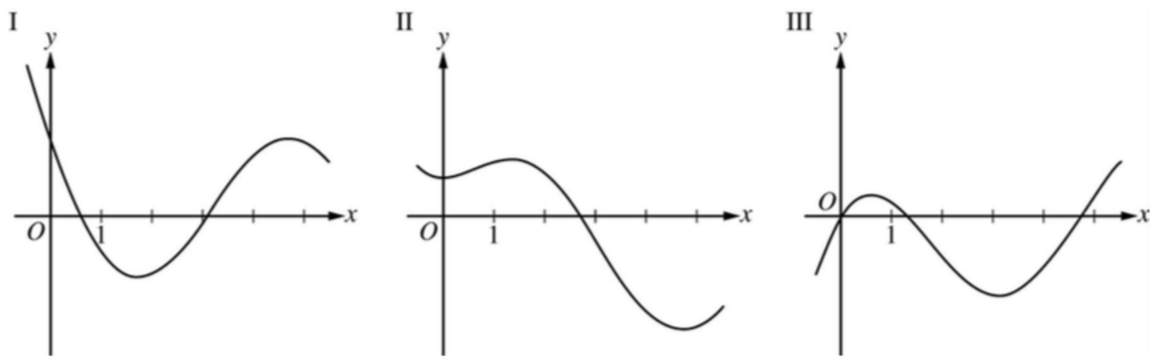
(A) $\dfrac{\sin(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$

(C) $\dfrac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\dfrac{1}{2x}\right)$

NC



4. Three graphs labeled I, II, and III are shown above. One is the graph of *f*, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	II	I	III
(C)	II	III	I
(D)	III	I	II

NC

5. The local linear approximation to the function *g* at $x = \frac{1}{2}$ is $y = 4x + 1$. What is the value of $g\left(\frac{1}{2}\right) + g'\left(\frac{1}{2}\right)$?

(A) 4

(B) 5

(C) 6

(D) 7

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NC 6. For time $t \geq 0$, the velocity of a particle moving along the x-axis is given by $v(t) = (t-5)(t-2)^2$. At what values of t is the acceleration of the particle equal to 0?

(A) 2 only

(B) 4 only

(C) 2 and 4

(D) 2 and 5

NC 7. The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

(A) The cost to shred 500 pounds of documents is \$80.

(B) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

(C) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately \$80.

(D) The cost to shred documents is increasing at a rate of \$80 per pound when the weight of the documents is 500 pounds.

NC 8. Which of the following integral expressions is equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\sqrt{1 + \frac{3k}{n}} \cdot \frac{1}{n} \right)$?

(A) $\int_0^1 \sqrt{1+3x}\, dx$

(B) $\int_0^3 \sqrt{1+x}\, dx$

(C) $\int_1^4 \sqrt{x}\, dx$

(D) $\frac{1}{3}\int_0^3 \sqrt{x}\, dx$

NC 9. $f(x) = \begin{cases} x & \text{for } x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

If f is the function defined above, then $\int_{-1}^{4} f(x)\, dx$ is

(A) $\frac{9}{2}$

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) undefined

NC

10. $\int e^x \cos(e^x + 1)\,dx =$

(A) $\sin(e^x + 1) + C$

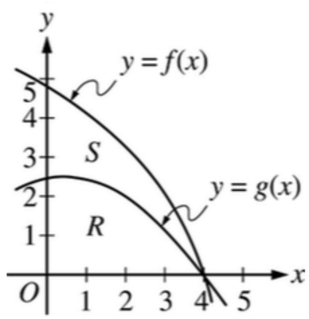
(B) $e^x \sin(e^x + 1) + C$

(C) $e^x \sin(e^x + x) + C$

(D) $\frac{1}{2}\cos^2(e^x + 1) + C$

Free Response (Goal: 15 min. each)

A graphing calculator is required for problems on this part of the exam.



1. Let R be the region in the first quadrant bounded by the graph of g, and let S be the region in the first quadrant between the graphs of f and g, as shown in the figure above. The region in the first quadrant bounded by the graph of f and the coordinate axes has area 12.142. The function g is given by $g(x) = (\sqrt{x+6})\cos\left(\frac{\pi x}{8}\right)$, and the function f is not explicitly given. The graphs of f and g intersect at the point $(4, 0)$.

   (A) Find the area of S.

   (B) A solid is generated when S is revolved about the horizontal line $y = 5$. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

   (C) Region R is the base of an art sculpture. At all points in R at a distance x from the y-axis, the height of the sculpture is given by $h(x) = 4 - x$. Find the volume of the art sculpture.

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2.

t (minutes)	0	3	5	6	9
$r(t)$ (rotations per minute)	72	95	112	77	50

Rochelle rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Rochelle's ride is modeled by a differentiable function r for $0 \le t \le 9$ minutes. Values of $r(t)$ for selected values of t are shown in the table above.

(A) Estimate $r'(4)$. Show the computations that lead to your answer. Indicate units of measure.

(B) Is there a time t, for $3 \le t \le 5$, at which $r(t)$ is 106 rotations per minute? Justify your answer.

(C) Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^9 r(t)\,dt$. Using correct units, explain the meaning of $\int_0^9 r(t)\,dt$ in the context of the problem.

(D) Sarah also rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Sarah's ride is modeled by the function s, defined by $s(t) = 40 + 20\pi \sin\left(\frac{\pi t}{18}\right)$ for $0 \le t \le 9$ minutes. Find the average number of rotations per minute of the wheel of the stationary bicycle for $0 \le t \le 9$ minutes.

Answers to Multiple-Choice Questions

1	C
2	B
3	B
4	C
5	D
6	C
7	D
8	A
9	B
10	A

Rubrics for Free-Response Questions

Question 1

Solutions	Point Allocation
(A) Area of region S = (Area under f) − (Area under g) $= 12.142 - \int_0^4 g(x)\,dx = 12.142 - 6.938$ $= 5.204$ (or 5.203)	3 : { 1 : integral; 1 : uses area under f; 1 : answer
(B) Volume $= \pi\int_0^4 \left((5 - g(x))^2 - (5 - f(x))^2\right) dx$	3 : { 2 : integrand; 1 : limits and constant
(C) Volume $= \int_0^4 ((4 - x)g(x))\,dx = 17.243$	3 : { 2 : integral; 1 : answer

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Question 2

Solutions	Point Allocation
(A) $r'(4) \approx \frac{r(5)-r(3)}{5-3} = \frac{112-95}{2} = \frac{17}{2}$ rotations per minute per minute	1 : answer with units
(B) r is differentiable $\Rightarrow$ r is continuous on $3 \le t \le 5$. $r(3) = 95 < 106 < 112 = r(5)$ Therefore, by the Intermediate Value Theorem, there is a time t, $3 \le t \le 5$, such that $r(t) = 106$.	$2 : \begin{cases} 1 : r(3) < 106 < r(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$
(C) $\int_0^9 r(t)\,dt \approx (3-0)\cdot r(0) + (5-3)\cdot r(3) + (6-5)\cdot r(5) + (9-6)\cdot r(6)$ $= 3(72) + 2(95) + 1(112) + 3(77) = 749$ $\int_0^9 r(t)\,dt$ is the total number of rotations of the wheel of the stationary bicycle over the time interval $0 \le t \le 9$ minutes.	$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$
(D) $\frac{1}{9}\int_0^9 s(t)\,dt = \frac{1}{9}\int_0^9 \left(40 + 20\pi \sin\left(\frac{\pi t}{18}\right)\right) dt$ $= \frac{1}{9}\left[40t - 360\cos\left(\frac{\pi t}{18}\right)\right]_0^9$ $= \frac{1}{9}\left(360 - 360\cos\left(\frac{\pi}{2}\right)\right) - \frac{1}{9}(0 - 360\cos(0))$ $= \frac{720}{9} = 80$ rotations per minute	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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