# Unit 5 Review

(AP-2012-No Calculator)

1. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.
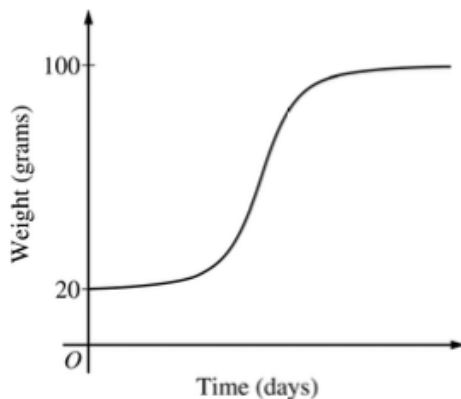
(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

2.

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(c) Draw the slope field for the 12 points shown in the graph.
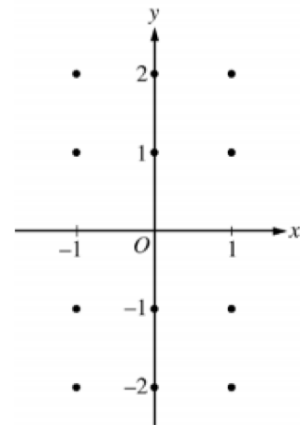
3. (Calculator allowed)
For the following, let S be the region enclosed by the graphs of
$y = \ln (x + 1)$, $x = 3$, and the x-axis.

a) Find the area of S.

b) Find the volume of the solid formed when S is rotated around the x-axis.

c) Find the volume of the solid formed when S is rotated around the line $y = 3$.

d) Find the volume of the solid formed when S is rotated around the line $x = -1$.

e) The region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle. Find the volume of this solid.

4. (Calculator allowed) For the following, let R be the region enclosed by the graphs of $y = \sin x$, $y = 1$, and the y-axis.

a) Find the area of R.

b) Find the volume of the solid formed when R is rotated around the x-axis.

d) Find the volume of the solid formed when R is rotated around the line $x = 2$.

e) Write the equations of axis of revolution that would require the disc method to find the volume of R rotated. There are two.

f) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 4 times the base. Find the volume of this solid.

5.
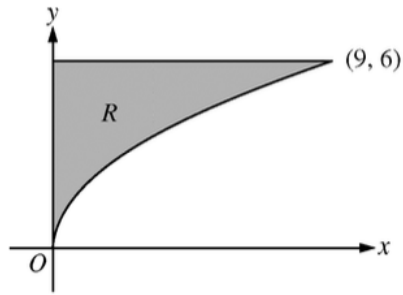
Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.