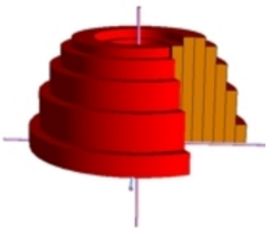
Name: Date:

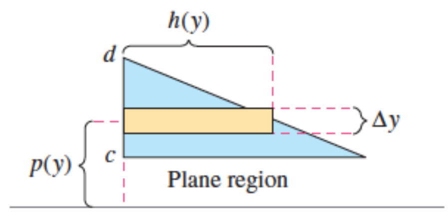
5D.1 Volume With the Shell Method



We have seen how the disk method and washer method can break down the volume of a solid of revolution into smaller parts that can be summed up using an integral. We will now apply a method that considers many hallow cylinders that make up the space occupied by a solid of revolution.

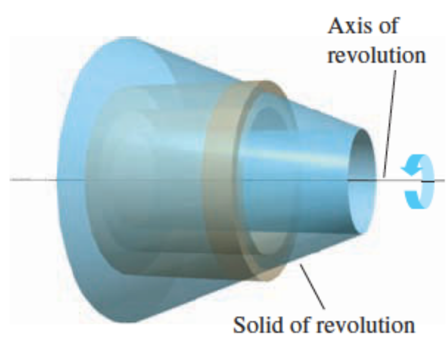
Consider the solid to the right with one "shell" drawn in. As $\Delta y \to 0$ the volume of the shell becomes equal to the lateral surface area of the cylinder.

$$Lateral\ Surface\ Area = circumference \cdot height$$
$$= (2\pi r)(height)$$
$$= 2p(y)h(y)$$



When we find the sum of the (infinitely thin) shells, we get

$$\int_c^d 2\pi p(y)h(y)\,dy$$

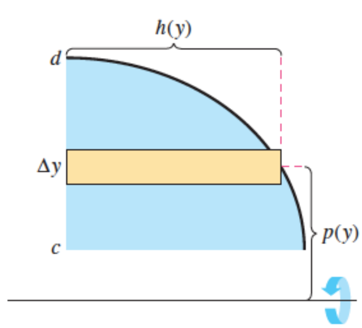


Where $p(y)$ is the radius and $h(y)$ is the height of the shell. Note that this is the volume of a solid revolved around the x −axis.

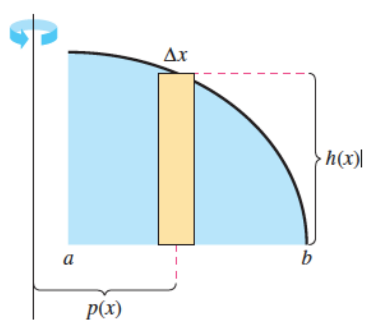
THE SHELL METHOD

To find the volume of a solid of revolution with the **shell method,** use one of the following, as shown in Figure 7.29.

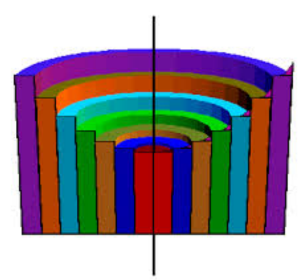
Horizontal Axis of Revolution	*Vertical Axis of Revolution*
Volume $= V = 2\pi \int_c^d p(y)h(y)\,dy$	Volume $= V = 2\pi \int_a^b p(x)h(x)\,dx$



Horizontal axis of revolution



Vertical axis of revolution

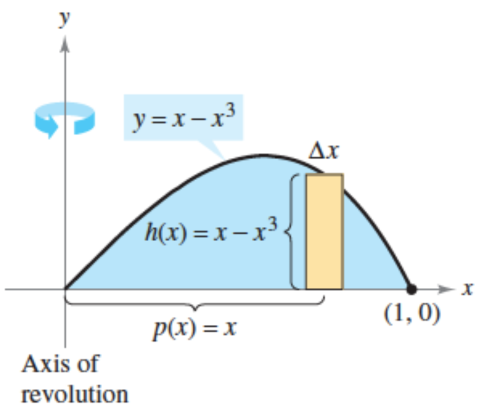


EXAMPLE 1 Using the Shell Method to Find Volume

Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = x - x^3$$

and the x-axis $(0 \le x \le 1)$ about the y-axis.

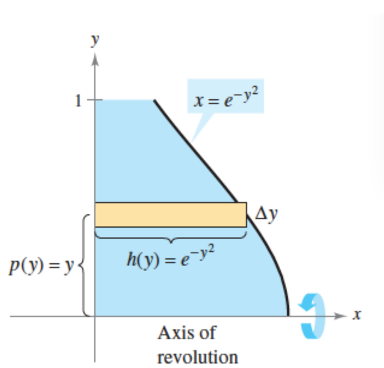


EXAMPLE 2 Using the Shell Method to Find Volume

Find the volume of the solid of revolution formed by revolving the region bounded by the graph of

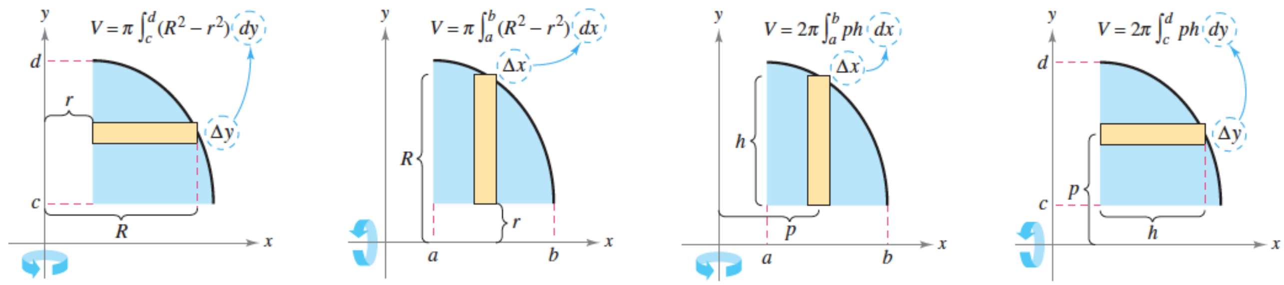
$$x = e^{-y^2}$$

and the y-axis $(0 \le y \le 1)$ about the x-axis.



Comparison of Disk and Shell Methods

The disk and shell methods can be distinguished as follows. For the disk method, the representative rectangle is always *perpendicular* to the axis of revolution, whereas for the shell method, the representative rectangle is always *parallel* to the axis of revolution, as shown in Figure 7.32.



Vertical axis of revolution — Horizontal axis of revolution

Disk method: Representative rectangle is perpendicular to the axis of revolution.

Vertical axis of revolution — Horizontal axis of revolution

Shell method: Representative rectangle is parallel to the axis of revolution.

Figure 7.32

Example 3

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$.