5B Exercises

In Exercises 1–8, verify the solution of the differential equation.

	Solution	*Differential Equation*
1.	$y = Ce^{4x}$	$y' = 4y$
5.	$y = C_1 \sin x - C_2 \cos x$	$y'' + y = 0$

In Exercises 9–12, verify the particular solution of the differential equation.

	Solution	*Differential Equation and Initial Condition*
9.	$y = \sin x \cos x - \cos^2 x$	$2y + y' = 2\sin(2x) - 1$, $y\left(\frac{\pi}{4}\right) = 0$

In Exercises 41–52, use integration to find a general solution of the differential equation.

41. $\frac{dy}{dx} = 6x^2$

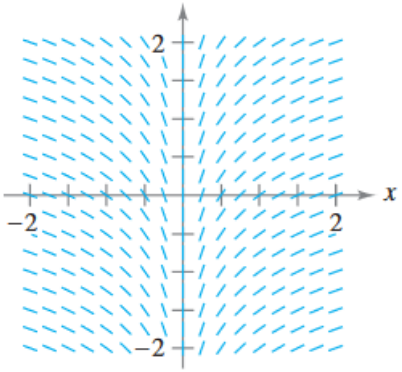
43. $\dfrac{dy}{dx} = \dfrac{x}{1 + x^2}$

49. $\dfrac{dy}{dx} = x\sqrt{x - 6}$

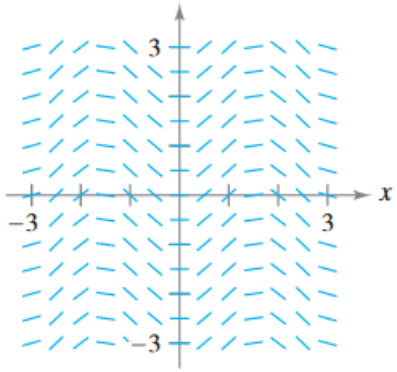
51. $\dfrac{dy}{dx} = xe^{x^2}$

In Exercises 57–60, match the differential equation with its slope field. [The slope fields are labeled (a), (b), (c), and (d).]

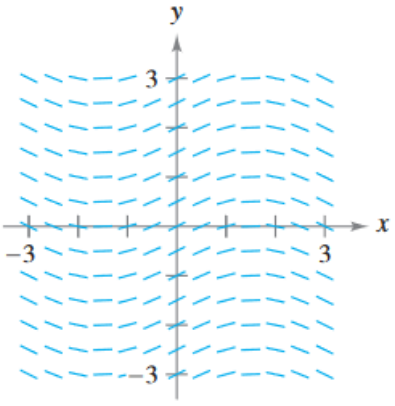
(a)



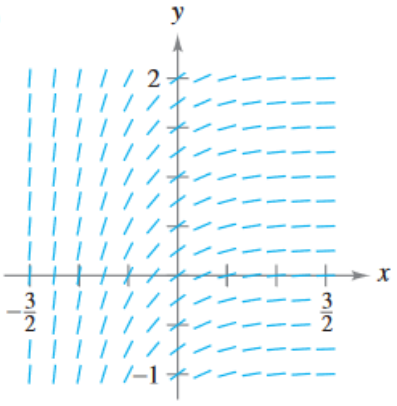
(b)



(c)



(d)



57. $\dfrac{dy}{dx} = \sin(2x)$

58. $\dfrac{dy}{dx} = \dfrac{1}{2}\cos x$

59. $\dfrac{dy}{dx} = e^{-2x}$

60. $\dfrac{dy}{dx} = \dfrac{1}{x}$

Differential Equations: AP Test Problems

1.

Let f be a function with $f(1)=4$ such that for all points (x,y) on the graph of f, the slope is given by $\frac{dy}{dx}=\frac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x=1$.

(b) Write an equation of the line tangent to the graph of f at $x=1$ and use it to approximate $f(1.2)$.

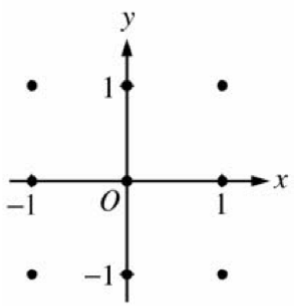
(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx}=\frac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

(d) Use your solution from part (c) to find the exact value of $f(1.2)$.

2.

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated. **(Note: Use the axes provided in the exam booklet.)**



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

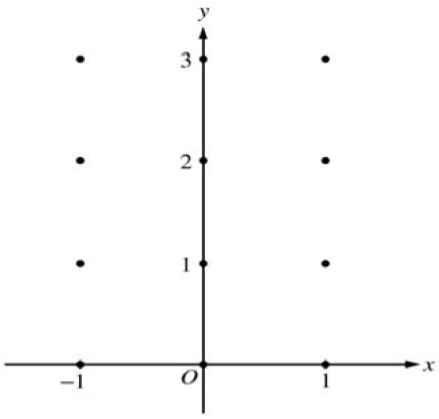
3.

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.



4. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

b) Find the domain and range of the function f found in part (a).