

Name: _____

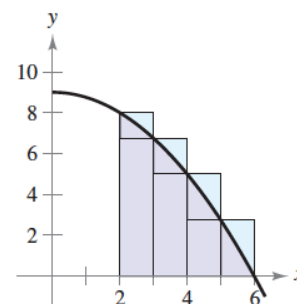
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Unit 4 Review

4A: Area and Integration (4.2)

1. Use the graph and 4 subintervals to approximate $\int_2^6 (9 - \frac{1}{4}x^2) dx$ using
 - a. Right-Hand Sum
 - b. Left-Hand Sum
 - c. Trapezoid Sum

$$y = 9 - \frac{1}{4}x^2$$



2. Write a summation to approximate the area between the graph of $y = 9 - \frac{1}{4}x^2$ and the x -axis using 20 rectangles with a right-hand sum. Then use your calculator to evaluate the approximation.
(Hint: you need to find the x -intercepts of $y = 9 - \frac{1}{4}x^2$ first)

4B: Riemann Sums (4.3)

3. Simplify the following summation

$$\sum_{i=0}^n (3i + 1)(i - 2)$$

For each of the following, (a) sketch a graph and shade the area that is represented by the integral, the (b) use the limit definition of an integral to evaluate the integral, then (c) use show how you can use FTC1 to check your answer.

4. $\int_0^3 (2x + 3) dx$

5. $\int_1^4 (10 - x^2) dx$

6. Use the First Fundamental Theorem of Calculus to evaluate.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(4 + \left(5 + \frac{2}{n} i \right)^5 \right) \left(\frac{2}{n} \right)$$

(Hint: this is a Riemann Sum that represents an integral)

7. Use the First Fundamental Theorem of Calculus to evaluate.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\sin \left(\frac{\pi}{2} + \frac{\pi}{n} i \right) \right) \left(\frac{\pi}{n} \right)$$

4C: Antiderivatives and Indefinite Integrals (4.1)

Find the indefinite integral.

8.

$$\int (4x^2 + x + 3) dx$$

9.

$$\int \frac{x^4 + 8}{x^3} dx$$

10.

$$\int \frac{2}{\sqrt[3]{3x}} dx$$

11.

$$\int (5 \cos x - 2 \sec^2 x) dx$$

12. Find the function $f(x)$ that passes through $(1, -2)$ if $f'(x) = -6x$

13.

Find the particular solution of the differential equation $f''(x) = 6(x - 1)$ whose graph passes through the point $(2, 1)$ and is tangent to the line $3x - y - 5 = 0$ at that point.

4D: The Fundamental Theorem of Calculus (4.4)

Use the First Fundamental Theorem of Calculus to evaluate

14.

$$\int_{-2}^{-1} (x^4 + 3x^2 - 4) dx$$

15.

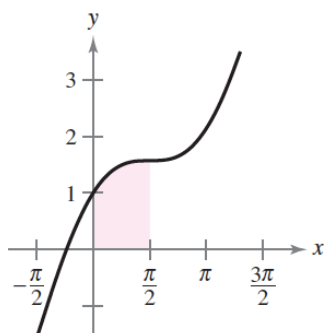
$$\int_4^9 x\sqrt{x} dx$$

16.

$$\int_{-\pi/4}^{\pi/4} \sec^2 t dt$$

17. Determine the area of the given region.

$$y = x + \cos x$$



18. Use the Second Fundamental Theorem of Calculus to find $F'(x)$

$$F(x) = \int_0^x t^2 \sqrt{1+t^3} dt$$

19. The velocity of a given function is given by the function $v(t) = (t-2)^2 - 1$. Find the displacement on the given intervals

a. $[0,6]$

b. $[2,3]$

c. What does the answer to b mean?

4E: Integration by Substitution (4.5)

Find the indefinite integral. Use u-substitution *if necessary*.

20.

$$\int \frac{x^2}{\sqrt{x^3+3}} dx$$

21.

$$\int \left(x + \frac{1}{x}\right)^2 dx$$

22.

$$\int x \sin 3x^2 dx$$

23.

$$\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x dx$$