Calculus

Name: Date:

4D.1: The 1st Fundamental Theorem of Calculus

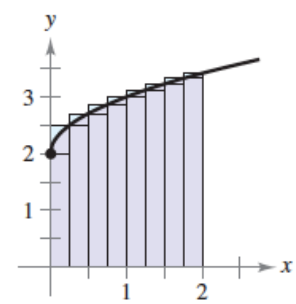
We now seek a more direct connection between the derivative and the integral operations. This brings us to the First Fundamental Theorem Calculus (which we refer to as FTC1).

First Fundamental Theorem of Calculus (FTC1)

If f is continuous on $[a, b]$ and F is the antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

A Quick Proof



Without going into too many confusing details, we can prove FTC1 using the mean value theorem. Consider any sub-interval that we are using to find the area under a curve.

The Mean Value Theorem says that on the sub-interval $[x_i, x_{i+1}]$, there exists a value c_i such that

$$F'(c_i) = \frac{F(x_{i+1}) - F(x_i)}{x_{i+1} - x_i}$$

Now we define $F'(c_i) = f(c_i)$, and we can let $\Delta x = x_{i+1} - x_i$. So, if we multiply by $(x_{i+1} - x_i)$ above we get

$$f(c_i)(x_{i+1} - x_i) = F(x_{i+1}) - F(x_i)$$
$$f(c_i)\Delta x = F(x_{i+1}) - F(x_i)$$

We have n sub-intervals from a to b. If we combine all of the sub-intervals, we get this

$$f(c_n)\Delta x + f(c_{n-1})\Delta x + \cdots + f(c_1)\Delta x = F(b) - F(x_{n-1}) + F(x_{n-1}) - F(x_{n-2}) + \cdots + F(x_1) - F(a)$$

Which we can simply write as

$$\sum_{i=1}^{n} f(c_i)\Delta x = F(b) - F(a)$$

At the limit as $n \to \infty$ we get

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

This is the Fundamental Theorem of Calculus!

FTC In Action

This gives us a slick way to find a definite integral without using limits and summations.

Use FTC to find the following definite integrals.

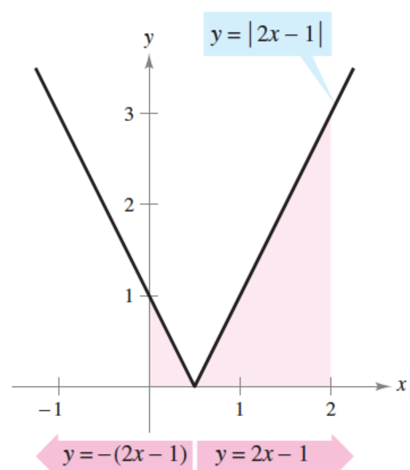
$$\int_1^2 (3x^2+1)\,dx$$

$$\int_1^4 2\sqrt{x}\,dx$$

$$\int_0^{\frac{\pi}{4}} \sec^2 x \; dx$$

Example 2 Split absolute values up to find the integral.

$$\int_0^2 |2x-1|\,dx$$



Example 3: Find the area between the graph of $y = 2x^2 - 3x + 2$ and the x-axis on the interval $[0,2]$.