4B Exercises

Extrema on an Interval

Evaluate the definite integral using the limit definition. Use $c_i = a + i\Delta x$ and $\Delta x = \frac{b-a}{n}$

3. $\int_2^6 8\,dx$ $\quad y = 8$ on $[2, 6]$. $\left(\textbf{Note: } \Delta x = \frac{6-2}{n} = \frac{4}{n}, \|\Delta\| \to 0 \text{ as } n \to \infty\right)$

$$\sum_{i=1}^{n} f(c_i)\Delta x_i = \sum_{i=1}^{n} f\left(2 + \frac{4i}{n}\right)\left(\frac{4}{n}\right) = \sum_{i=1}^{n} 8\left(\frac{4}{n}\right) = \sum_{i=1}^{n} \frac{32}{n} = \frac{1}{n}\sum_{i=1}^{n} 32 = \frac{1}{n}(32n) = 32$$

$$\int_2^6 8\,dx = \lim_{n\to\infty} 32 = 32$$

4. $\int_{-2}^{3} x\,dx$

$$\Delta x = \frac{3-(-2)}{n} = \frac{5}{n}; c_i = -2 + \frac{5}{n}i$$

$$\sum_{i=1}^{n}\left(\left(-2 + \frac{5}{n}i\right)\left(\frac{5}{n}\right)\right) = \sum_{i=1}^{n}\left(-\frac{10}{n} + \frac{25}{n^2}i\right) = \sum_{i=1}^{n}\left(-\frac{10}{n}\right) + \sum_{i=1}^{n}\left(\frac{25}{n^2}i\right) = -10 + \frac{25}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= -10 + \frac{25n^2 + 25}{2n^2}$$

$$\lim_{n\to\infty}\left(-10 + \frac{25n^2 + 25}{2n^2}\right) = -10 + \frac{25}{2} = 2.5$$

6. $\int_1^4 4x^2\,dx$

$$\Delta x = \frac{4-1}{n} = \frac{3}{n}; c_i = 1 + \frac{3}{n}i$$

$$\sum_{i=1}^{n} f\left(1 + \frac{3}{n}i\right)\left(\frac{3}{n}\right) = \sum_{i=1}^{n}\left(4\left(1 + \frac{3}{n}i\right)^2\right)\left(\frac{3}{n}\right) = \sum_{i=1}^{n}\left(4\left(1 + \frac{6}{n}i + \frac{9}{n^2}i^2\right)\right)\left(\frac{3}{n}\right) =$$

$$\sum_{i=1}^{n}\left(\frac{12}{n} + \frac{72}{n^2}i + \frac{108}{n^3}i^2\right) = 12 + \frac{72}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{108}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 12 + \frac{36n^2 + 36n}{n^2} + \frac{72n^3 + 54n^2 + 18n}{n^3}$$

$$\lim_{n\to\infty}\left(12 + \frac{36n^2 + 36n}{n^2} + \frac{72n^3 + 54n^2 + 18n}{n^3}\right) = 12 + 36 + 72 = 120$$

7. $\int_1^2 (x^2 + 1)\, dx$

$y = x^2 + 1$ on $[1, 2]$. $\left(\textbf{Note: } \Delta x = \dfrac{2-1}{n} = \dfrac{1}{n},\ \|\Delta\| \to 0 \text{ as } n \to \infty\right)$

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i = \sum_{i=1}^{n} f\left(1 + \frac{i}{n}\right)\left(\frac{1}{n}\right)$$

$$= \sum_{i=1}^{n}\left[\left(1 + \frac{i}{n}\right)^2 + 1\right]\left(\frac{1}{n}\right)$$

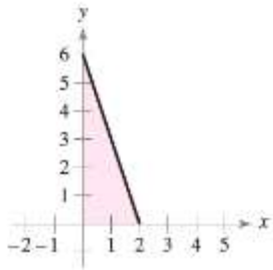
$$= \sum_{i=1}^{n}\left[1 + \frac{2i}{n} + \frac{i^2}{n^2} + 1\right]\left(\frac{1}{n}\right)$$

$$= 2 + \frac{2}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2 = 2 + \left(1 + \frac{1}{n}\right) + \frac{1}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) = \frac{10}{3} + \frac{3}{2n} + \frac{1}{6n^2}$$

$$\int_1^2 \left(x^2 + 1\right) dx = \lim_{n\to\infty}\left(\frac{10}{3} + \frac{3}{2n} + \frac{1}{6n^2}\right) = \frac{10}{3}$$

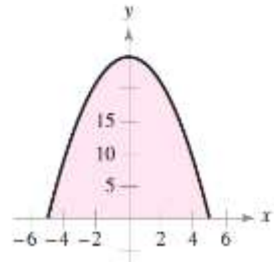
Write a definite integral that defines the area of the region (don't evaluate)

14. $f(x) = 6 - 3x$



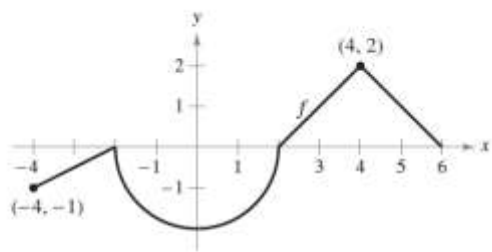
$$\int_0^2 6 - 3x \; dx$$

$f(x) = 25 - x^2$



$$\int_{-5}^{5} (25 - x^2) \, dx$$

47. ***Think About It*** The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.



(a) $\int_0^2 f(x)\,dx$ (b) $\int_2^6 f(x)\,dx$ (c) $\int_{-4}^2 f(x)\,dx$

(d) $\int_{-4}^6 f(x)\,dx$ (e) $\int_{-4}^6 |f(x)|\,dx$ (f) $\int_{-4}^6 [f(x) + 2]\,dx$

(a) Quarter circle below x-axis:
$-\frac{1}{4}\pi r^2 = -\frac{1}{4}\pi(2)^2 = -\pi$

(b) Triangle: $\frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$

(c) Triangle + Semicircle below x-axis:
$-\frac{1}{2}(2)(1) - \frac{1}{2}\pi(2)^2 = -(1 + 2\pi)$

(d) Sum of parts (b) and (c): $4 - (1 + 2\pi) = 3 - 2\pi$

(e) Sum of absolute values of (b) and (c):
$4 + (1 + 2\pi) = 5 + 2\pi$

(f) Answers to (d) plus
$2(10) = 20$: $(3 - 2\pi) + 20 = 23 - 2\pi$